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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LXXVI

JULY 1902

Frontispiece

On Magnetostriction in Bismuth — A. P. WILLS — 1

Contributions to the Study of the Induction Coil, II. — J. E. IVES — 7

Viscosity of Liquids determined by Measurement of Capillary Waves — P. R. WATSON — 20

Wireless Telegraphy with a Relay-Telephone Receiver — A. H. TAYLOR — 39

The Vowel I^{e} (as in Pit) — LOUIS BEVIER, Jr. — 44

Peter Guthrie Tait — ALEXANDER MACFARLANE — 51

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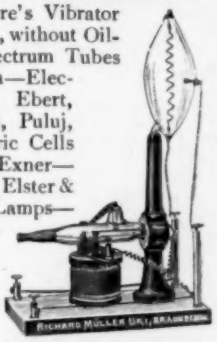

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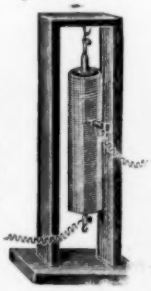

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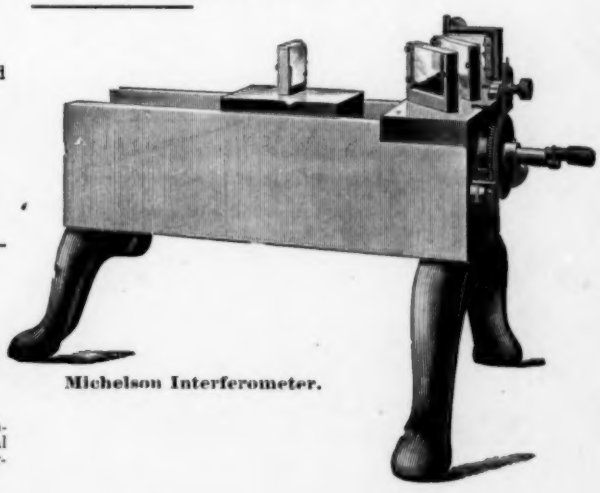

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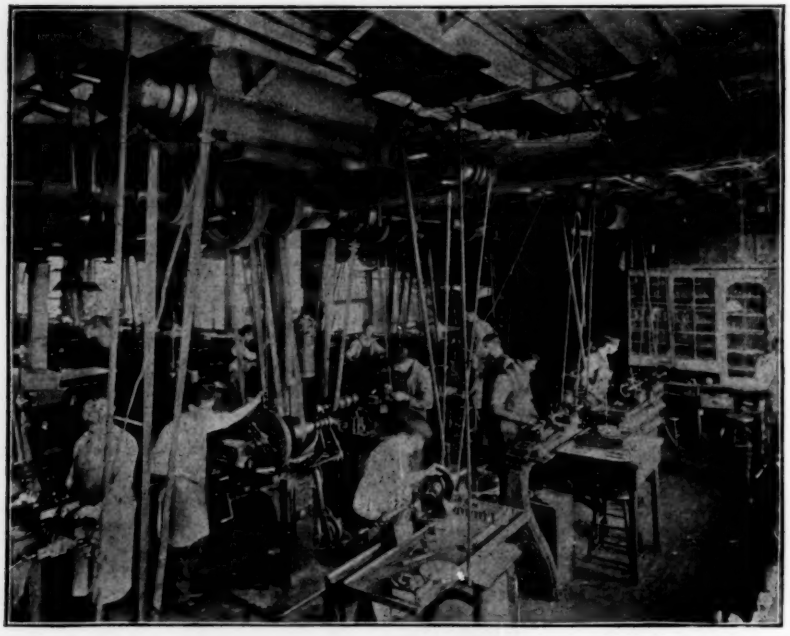

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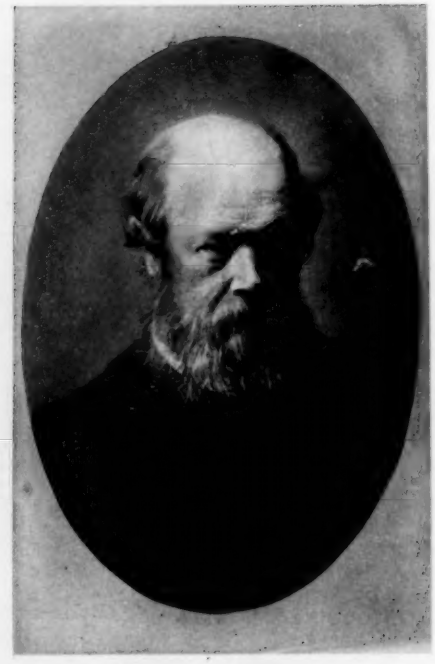

P. G. Tait.

Volume XV. *July, 1902.* *Number 1.*

THE

PHYSICAL REVIEW.

ON MAGNETOSTRICTION IN BISMUTH.

By A. P. Wills.

IN his extensive paper on magnetostriction in iron and some other metals, Bidwell [1] describes a remarkable effect which he observed on magnetizing a bismuth rod. He employed a cast cylindrical rod 13.2 cm. in length and .7 cm. in diameter. It was magnetized lengthwise by a powerful solenoid. With a magnetizing force of 280 C. G. S. units an elongation was suspected; with 470 units it was quite perceptible. Upon increasing the field to 680 units the extension was roughly measurable and amounted to .3 ten millionths of the length of the rod; and with a field of 842 units the extension was 1.5 ten millionths of the length. From these results it appears that the effect was increasing much more rapidly than the field. The field was not increased beyond the last-mentioned figure.

Since bismuth is the only substance outside of the strictly magnetic metals in which this effect has been observed, it seemed desirable to study the behavior of bismuth in this connection under the action of much higher fields. Now in the case of bismuth an electromagnet can be advantageously employed. For, a short rod can be used as well as a long one, since the end effects can be neglected and the pole pieces can therefore be brought close enough together to produce the powerful field desired.

[1] Philosophical Transactions, Vol. 179, p. 205.

With this idea in mind I wrote to Hartmann and Braun of Frankfurt am Main for samples of their electrolytically prepared bismuth. They furnished these in the form of cast cylinders somewhat over 1 cm. in length and .5 cm. in diameter. I turned one of these until its length was 1.1 cm. and its diameter .52 cm. The main experiments were performed with this cylinder.

In the preliminary experiments, I had at my disposal a very powerful electromagnet belonging to the physics department of the Massachusetts Institute of Technology.[1] Later, at Bryn Mawr, I used a much smaller one.

The arrangement for measuring any possible change in length of the bismuth cylinder consisted of a system of two levers and a high power micrometer microscope.

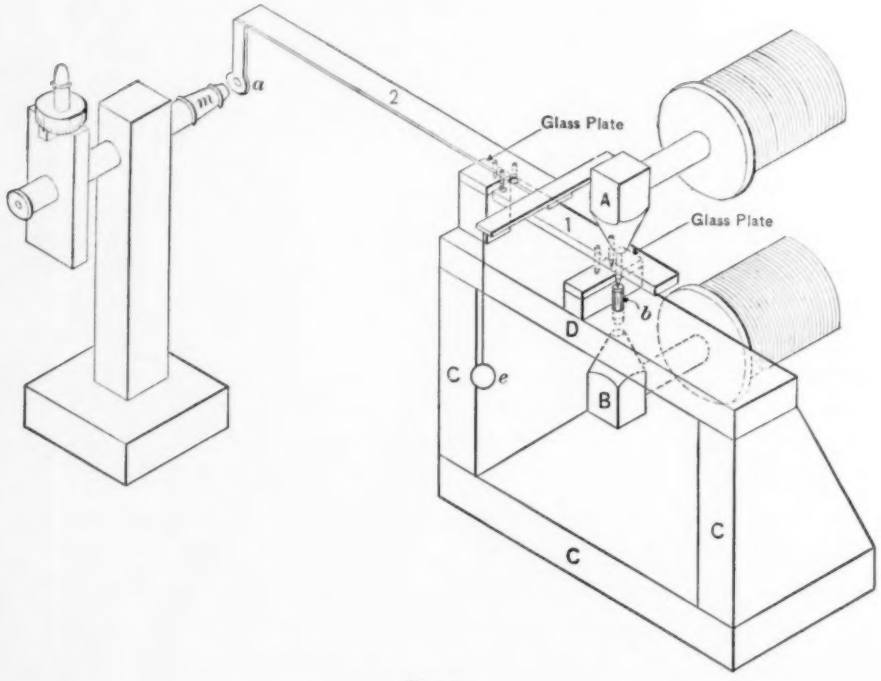

Fig. 1.

In Fig. 1 A and B are the pole pieces of the electromagnet; b is the bismuth cylinder; 1 and 2 are the two levers and m is the

[1] I wish to acknowledge here the courtesy of Professor Cross and Professor Wendell in putting the apparatus of the physical department at my disposal.

micrometer microscope; *a* is a portion of a microscope slide containing diatoms (upon one of which the microscope was focused): *C* is a heavy framework of wood and *D* is a heavy bar of brass. A circular recess was made at about the middle of this bar of such diameter that it made an easy sliding fit for the cylinder. The bottom of this recess was plane, so that the cylinder rested firmly upon it. A small bit of plane glass was shellacked to the upper face of the cylinder. Each of the levers played on three points. These points were made of sapphire and were highly polished. The levers themselves were of aluminium and quite light. One point on the first lever pressed against the glass on the end of the bismuth cylinder and the other two against a horizontal plate of glass firmly fixed to the supporting bar *D*. These three points were held properly in place by a small spring (not shown in the figure) pulling downward from a point midway between them. The tension on this spring could be varied at pleasure by turning a small windlass to which it was connected. At one end of the first lever there were two sapphire jewels firmly attached with shellac. One of these was a "V" jewel, the other a "slot" jewel. In these two of the points of the second lever played while the third rested on a glass plate firmly fixed to the bar *D*. There was a counterweight *d* and another (not shown in the figure) symmetrically placed which served to bring the center of gravity of the second lever in a vertical line about midway between the points. When in use, a box to protect the second lever from air draughts was found very desirable. All of the supports except that for the electromagnet were placed upon a brick pier. The support for the electromagnet was upon a table of its own, so that there was no material contact between the electromagnet and the apparatus on the pier. This was desirable, since the magnet supports would give slightly when the field was thrown on and any contact might cause a disturbance in the delicate lever system.

The lengths of the arms of the two levers and their multiplying powers were as follows:

	Short Arm.	Long Arm.	Multiplying Power.
Lever 1.	1.10 cm.	12.35 cm.	11.23.
Lever 2.	.41 cm.	24.0 cm.	48.4.

Hence the multiplying power of both together was $11.23 \times 58.5 = 657$. It was found that 1,162 head divisions on the micrometer eye-piece represented a length of one-tenth of a millimeter. Hence one head division would correspond to a change in length in centimeters of the bismuth cylinder equal to

$$\frac{1}{1162 \times 100 \times 657} = 1.31 \times 10^{-8}.$$

Now the maximum extension observed by Bildwell amounted to 1.8 ten-millionths of the length of the rod which was 13.2 cm. Hence the extension per centimeter of length would be 1.36×10^{-8} cm., and, in the present experiments, with a field of 1,000 an extension should have been detected.

In the experiments tried the field was varied, reaching a maximum of 3,200 C.G.S. units; but neither with the maximum field nor with any of the various intermediate fields could any effect be detected which was not due to one of the following causes: (1) A heat effect, in which case time was required to produce it. (2) An effect due to the action of the magnet on the currents induced in the levers themselves; in this case a sudden jerk was observed when the magnet circuit was made or broken and then the pointer would return to its zero position immediately. This effect was much more marked when the circuit was broken than when made.

These experiments was repeated with a bismuth cylinder made of bismuth which one buys in the market as chemically pure. But again negative results only were obtained.

After working with levers for some time it became evident that great care must be used if their action was to be relied on. The first system constructed was wrong somewhere as it was suspiciously non-susceptible to small vibrations and to small variations in temperature. The second lever was quite heavy and I suspected that the points were not right. So I made a new lever much lighter than the old and gave the points the arrangement shown in Fig. 1, except for the "V" and the "slot" jewels. Finally, however, I tried the point, slot, plane arrangement of the points and this im-

proved the action considerably, adding to the stability of the system without interfering with its sensitiveness. But in order to have a check upon the sensitiveness of the levers it was decided to measure the depression of a stiff steel spring under the action of small weights.

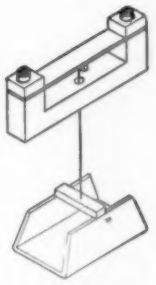

Fig. 2.

The spring was clamped firmly at both ends to a support (see Fig. 2) which was so designed that, when the bismuth cylinder was removed, it could be screwed to the brass bar D in such a way that the point of the first lever which rests on the cylinder could be made to rest on a bit of glass shellacked to the spring in the center. Below this point a string passed through holes in the supports and at the end of the string a stirrup for weights was attached.

The dimensions of the spring were as follsws :

Length,	2.70 cm.
Width,	.59 "
Depth,	.05 "

The theoretical depression of the center of the spring under known loads could be calculated from the formula for the case of a prismatic rod clamped at both ends with a load in the middle. In the calculations 23×10^{11} was assumed as the value of Young's modulus. This is roughly the mean of the values found in the tables. With this assumption a load of one gram should give a depression of 7.05×10^{-6} cm. But since there was a narrow strip of brass (for attaching the stirrup string) soldered transversely to the spring this value is a little larger than it really ought to be. The depression of the spring as actually measured was 6.70×10^{-6} cm. The agreement is quite satisfactory. Furthermore the depression was found to be proportional to the load. This seems good evidence that the lever system finally worked as it should. Further evidence is afforded by the fact that when the bismuth cylinder was in position and a lighted match held in its neighborhood the pointer in the microscope moved gradually and not jerkily.

The field was measured ballistically by means of a d'Arsonval galvanometer. The galvanometer was calibrated by means of a long standardizing solenoid.

It is intended to continue these experiments soon with a much more powerful electromagnet.

PHYSICAL LABORATORY, BRYN MAWR COLLEGE,
March 14, 1902.

CONTRIBUTIONS TO THE STUDY OF THE INDUCTION COIL. II.

By James Edmund Ives.

§ 7. Sparking at the Break.

Perhaps the subject of greatest practical importance at the present time in connection with the induction coil, is the sparking across the break, its cause and its cure. Fizeau showed in 1853 that a condenser when placed around the break reduced the sparking, but no complete theory of its action has yet been given. The question "How much capacity must be put around the break, for any given induction coil, to stop the sparking?" remains still unanswered. One of the objects of this investigation was to solve this interesting problem.

Mizuno,[1] Johnson[2] and others have shown definitely that after a certain amount of capacity has been put around the break, the secondary potential, instead of being increased by further increase of the capacity, is decreased.

When the secondary of an induction coil is open, the primary behaves practically as though the secondary were not present, and the problem reduces to the problem of the single circuit which has been treated in § 2. For the sake of simplicity then, we shall treat the action of the primary as though it were uninfluenced by the presence of the secondary.

When a circuit, containing an impressed electromotive force, an inductance, and a capacity around the break, is broken what happens is apparently as follows. At the instant of breaking the circuit the current has a certain steady value. When the circuit is broken the current falls very rapidly to its zero value. In virtue of the capacity and inductance of the circuit, the current does not immediately

[1] Phil. Mag., Vol. 45, pp. 447–454, 1898.
[2] Phil. Mag., Vol. 49, pp. 216–220, 1900.

vanish, but oscillates about its zero value, with a period which is given approximately by

$$T = 2\pi \sqrt{LC}.$$

If the capacity of the condenser is very small the rate of decrease of the current is very great, and a high induced electromotive force is generated in the primary, which is great enough to disintegrate (ionize?) the dielectric between the poles of the break, and cause a current to flow across it. This current we may call the "leakage current."

The leakage current immediately reduces the rate of fall of the primary current, and since the induced electromotive force in the secondary is proportional to the rate of fall of the primary current, reduces the difference of potential in the secondary. It is therefore evident that in order to produce the maximum electromotive force in the secondary the rate of fall of the primary should be made as great as possible, without making it so great that the induced electrometer force in the primary produces a leakage current across the break. This is done by putting in capacity around the break until the rate of fall of the primary current has been sufficiently decreased.

Since some dielectrics break down less readily than others, the rate of fall of the primary current can be made greatest when the dielectric having the greatest breaking strength is used. Thus for instance kerosene and alcohol have a greater breaking strength than air, and when the break of an induction coil is made in these liquids, the electromotive force in the secondary is much greater than when it is made in air.

The oscillations of the primary current have been carefully investigated by Walter[1] with a Braun tube. His observations were found to agree with the theory.

The amount of capacity around the break which gives the greatest difference of potential in the secondary has been called by Johnson[2] the *optimum capacity*.

It is now proposed to find a mathematical expression for this optimum capacity.

[1] Wied. Ann., Bd. 62, pp. 307–316, 1897.
[2] Loc. cit., p. 220.

It is evident that the sparking across the break is due to the difference of potential between the two sides of the condenser, since the condenser is around the break. It is also evident that when $t = 0$, that is at the instant when the break occurs, there is no difference of potential between the two sides of the condenser. The growth of the potential of the condenser is given by equation (11). It is proportional to a sine function of the time. The potential of the condenser therefore does not rise instantly. If the movable contact of the break can get away fast enough so that the potential has not had time to rise to the value necessary to perforate the intervening dielectric, there will be no sparking. In order that there shall be no sparking, the time derivative of the condenser potential must be less than or at the most equal to the time derivative of the potential necessary to produce rupture. Writing

V_c = potential of the condenser,

V_r = potential necessary to produce rupture,

we have as the necessary and sufficient condition that there shall be no sparking

$$\left(\frac{dV_c}{dt}\right)_{t=0} \leqq \left(\frac{dV_r}{dt}\right)_{t=0}. \tag{36}$$

Since the interval of time under consideration is very small the damping may be neglected and equation (11) becomes

$$V_c = \frac{I_0}{K\nu} \sin \nu t.$$

For which we get

$$\frac{dV_c}{dt} = \frac{I_0}{K} \cos \nu t,$$

and

$$\left(\frac{dV_c}{dt}\right)_{t=0} = \frac{I_0}{K}. \tag{37}$$

For very thin layers of the dielectric, V_r, the rupture potential is found to be a linear function of their thickness, *i. e.*,

$$V_r = ax,$$

where a is the *rupture coefficient* and x the distance between the fixed and movable contacts.

We have therefore

$$\frac{dV_r}{dt} = a\frac{dx}{dt} = av$$

where v is the velocity of the moving contact, and

$$\left(\frac{dV_r}{dt}\right)_{t=0} = av_0 \tag{38}$$

where v_0 is the velocity of the moving contact when $t = 0$.

Equating (37) and (38) we have

$$\frac{I_0}{K} = av_0,$$

i. e.,

$$K = \frac{I_0}{av_0}. \tag{39}$$

That is, K, the optimum capacity, is directly proportional to the steady current, and inversely proportional to the velocity of breaking, and to the dielectric constant.

It is a rather curious result, that it seems to be independent of the inductance of the circuit. This arises from the fact that the rate of growth of the potential is taken at the time when $t = 0$, and is for that particular value of t independent of the inductance.

Equation (39) appears to indicate that for the same current and the same dielectric the optimum capacity depends only upon the rate of breaking of the primary circuit.

Equation (39) may be written

$$v_0 = \frac{I_0}{aK}, \tag{40}$$

which shows that if K is any capacity around the break, in order that there shall be no sparking, v_0 must have the value given by this equation. v_0 may therefore be called the *safe velocity*, that is, that velocity which is both necessary and sufficient in order that there shall be no sparking.

If equation (40) be true, when we vary the speed of breaking, without changing the capacity, the secondary potential should increase up to a certain point and then remain constant. The velocity

at which this occurs for a given capacity, is what we have called the "safe velocity"; it might be called the "optimum velocity" for this capacity.

A number of experiments were made in the endeavor to show that such a velocity existed. The results obtained were not very definite.

Some of the results are given in Tables XV.–XVII., and are plotted in Figs. 16–18. The velocity of breaking the primary varied from 70 cm. per sec. to zero velocity. The primary current, which was kept constant during each experiment, varied from one to three ampères.

These experiments show a tendency towards a decrease in the secondary potential as the velocity of breaking is decreased, but do not as a rule, show any well marked value of the velocity above which the potential remains constant, as would be expected from the theory. Some of the experiments however seem to indicate such a point: for example Exps. 1 and 2 in Table XV.; Exp. 3, Table XVI.; and Exps. 1, 4 and 5, Table XVII. A serious difficulty is the lack of consistency in the experimental results. Sparking seems to be a variable and erratic phenomenon. The results seem to show an unstable condition for very small velocities, so that very different results are liable to be obtained under what appear to be exactly the same experimental conditions. The mean depression of the potential in the neighborhood of zero velocity is not very great, amounting to only 10 or 20 per cent. of the whole potential.

The breaking contacts used in these experiments were platinum.

Above velocities of 10 cm. per second, the potential remained practically constant, and so far as could be determined no change was produced on increasing the velocity above this point.

The experiments were performed with three different kinds of breaks.

In the first break, which may be called the *guillotine break*, a heavy body, such as a piece of lead, falls in guides, and in falling strikes a little lever which breaks the current. The height through which the body falls determines the velocity, neglecting the friction. The lever was elbow-shaped, and similar to the one described below

in the case of the *rotating break*. The results obtained with the guillotine break are given in Table XV. and Fig. 16.

TABLE XV.

(*Guillotine Break.*)

Exp. 1. $K_1 = 0$ $I_0 = 2.5$ Amp.		Exp. 2. $K_1 = .25$ Mf. $I_0 = 2.0$ Amp.		Exp. 3. $K_1 = .25$ Mf. $I_0 = 3.2$ Amp.		Exp. 4. $K_1 = 1.07$ Mf. $I_0 = 1.0$ Amp.		Exp. 5. $K_1 = 1.07$ Mf. $I_0 = 2.5$ Amp.	
Velocity of Break in Cm. Per. Sec.	Mean Potential in Volts.	Velocity.	Potential.	Velocity.	Potential.	Velocity.	Potential.	Velocity.	Potential.
69	10.1	69	5.6	69	13.9	69	5.4	69	11.6
61	10.0	13	5.8	61	14.1	25	5.8	61	11.6
51	9.9	11	5.7	51	14.1	20	5.8	51	11.6
39	10.0	7.3	5.6	39	14.0	13	5.3	39	11.7
20	10.0	5.0	4.8	29	13.7	11	5.0	20	11.6
13	9.8	3.0	4.5	20	14.1	7.3	5.6	13	11.2
9.3	9.5	0.0	4.8	13	13.7	5.0	5.2	5.0	10.5
6.3	9.2			11	13.9				
5.0	8.0			7.3	13.5				
0.0	7.2			5.0	13.4				

The second break may be called the *free fall break*, since the circuit is broken by a body falling freely in air. The instrument used was the drop chronograph constructed by Professor Webster for his experiments on electrical oscillations,[1] and already described by him. The falling body was a steel cylinder, containing mercury. The little lever which the body strikes in falling is elbow-shaped and similar to the one described below in the case of the rotating break. The small distances fallen through were measured with a micrometer screw. The results obtained with the free fall break are given in Table XVI. and Fig. 17.

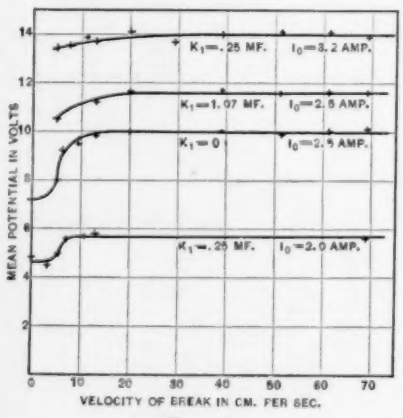

Fig. 16.

[1] PHYS. REV., Vol. VI., pp. 298–300, 1898.

TABLE XVI.

(*Free Fall Break.*)

Exp. 1. $K_1 = 0$ $I_0 = 2.0$ Amp.		Exp. 2. $K_1 = .102$ Mf. $I_0 = 2.0$ Amp.		Exp. 3. $K_1 = .25$ Mf. $I_0 = 2.0$ Amp.		Exp. 4. $K_1 = .25$ Mf. $I_0 = 2.0$ Amp.		Exp. 5. $K_1 = .25$ Mf. $I_0 = 4.0$ Amp.	
Velocity of Break in Cm. Per Sec.	Mean Potential in Volts.	Velocity.	Potential.	Velocity.	Potential.	Velocity.	Potential.	Velocity.	Potential.
19.8	6.3	24.3	5.7	19.8	5.9	9.9	5.3	14.0	7.8
14.0	6.3	19.8	5.7	14.0	5.9	4.4	5.0	9.9	8.1
9.9	6.3	14.0	5.9	9.9	5.8	4.2	5.2	6.2	7.7
7.0	6.2	9.9	5.8	7.0	6.0	3.9	5.2	5.4	7.5
6.2	6.1	8.9	5.3	5.4	5.8	3.7	5.1	4.4	7.6
5.4	6.0	7.7	5.5	4.4	5.8	3.4	4.9	3.7	7.3
4.4	5.5	6.2	5.6	3.9	5.5	3.1	4.9	3.4	7.9
2.8	5.2	5.4	5.3	3.7	5.6	0.0	4.5	3.1	7.6
2.4	5.7	4.4	5.1	3.4	4.8			2.8	7.7
2.0	5.6	4.2	5.1	3.1	4.9			2.4	7.3
1.4	5.1	3.9	5.1	2.8	4.4			2.0	7.4
0.0	5.4	3.4	5.1	2.4	4.9			1.4	7.1
		3.1	5.3	2.0	4.8			0.0	7.5
		1.4	5.5	1.4	4.7				
		0.0	4.9	0.0	4.8				

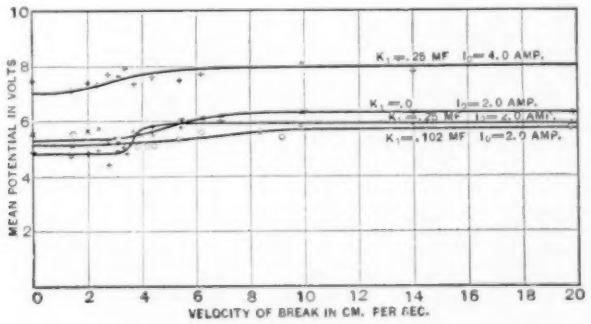

Fig. 17.

The difficulty of measuring the small distance fallen through with accuracy led to the use of the third kind of break, which may be called the *rotating break*. In this break a cam was attached rigidly to a heavy wheel or pulley, which by suitable gearing could be made to rotate in any desired period by means of a small electric motor. The motor had an automatic governor on it to keep its speed constant. The cam in the course of a rotation struck one

limb of a small steel lever, made in the form of an elbow. On the end of the other limb was a plane platinum surface about 3 mm. in diameter, which when the circuit was closed was applied to a similar platinum surface, which was fixed in position. When the circuit was closed, the surfaces were made to press against each other by an elastic band. The small pressure of the elastic band was found to be sufficient to keep the current constant. The results for this break are given in Table XVII. and Fig. 18.

TABLE XVII.

(*Rotating Break.*)

Exp. 1. $K_1 = .25$ Mf. $I_0 = 2.0$ Amp.		Exp. 2. $K_1 = .25$ Mf. $I_0 = 2.0$ Amp.		Exp. 3. $K_1 = .25$ Mf $I_0 = 2.0$ Amp.		Exp. 4. $K_1 = 5$ Mf. $I_0 = 2.2$ Amp.		Exp. 5. $K_1 = .5$ Mf. $I_0 = 2.2$ Amp.	
Velocity of Break in Cm. Per Sec.	Mean Potential in Volts.	Velocity.	Potential.	Velocity.	Potential.	Velocity.	Potential.	Velocity.	Potential.
36.3	4.9	36.3	5.7	19.0	5.2	21.0	5.7	20.3	5.1
15.3	4.8	12.0	5.4	8.7	5.6	9.0	5.6	10.0	5.2
13.0	4.5	3.2	4.8	3.3	5.4	2.3	5.3	5.0	5.4
9.3	4.8			3.1	5.5	2.1	4.9	2.5	4.6
6.5	3.9			.2	4.9	2.0	5.1	1.0	4.5
.9	4.8					1.0	4.8	.3	4.7
						.4	5.1		
						.3	4.9		

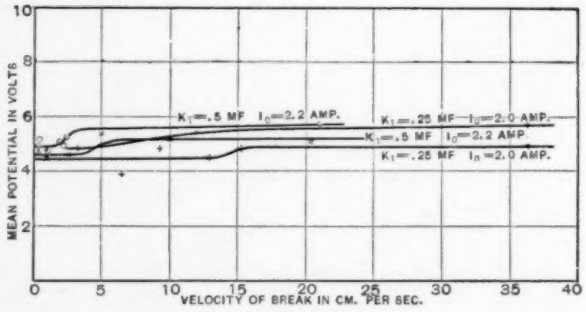

Fig. 18.

In each case the difference of potential in the secondary was measured by using the electrometer *ballistically*, that is by observing the throw of the needle on breaking the circuit. This is evidently proportional to the mean square of the difference of potential during the short period of the oscillation of the current in the primary.

In these experiments the primary and secondary coils already described were used, with no iron in the core. Since L_1 was equal to about .0003 of a henry, and R_1 equal to .66 of an ohm, for K_1 equal to a microfarad, the period of oscillation was given by $T_1 =$ .00011 sec., and the time constant by $\tau =$ .0009 sec. For $K_1 =$ .5 Mf., $T_1 =$ .00008 sec., and for $K_1 =$ .25 Mf., $T_1 =$ .00006 sec.

In order to calculate v_0 it is necessary to know a. The latest work on sparking distances is by Mr. R. F. Earhart.[1] His determinations were made for brass surfaces, one surface convex and the other plane. From the curve given by him a can be determined. For a distance apart of from 0 to 3 microns its value is 1,500,000 volts per cm. Its value then changes abruptly, and from 3 to 100 microns becomes equal to 70,000 volts per cm. Since we wish the value of a at the instant of breaking we shall take the larger value.

Putting in the value of I_0, a, and K in (40) we get :

For $K =$ 1.0 Mf. $v_0 =$ 1.34 cm. per sec.
For $K =$ .5 Mf. $v_0 =$ 2.67 cm. per sec.
For $K =$ .25 Mf. $v_0 =$ 5.34 cm. per sec.

Therefore the safe velocity, if it exists, lies well within the range of the experiments, and its existence should be shown by them.

Only two experiments were made keeping the velocity constant and changing the capacity, as the ballistic method of using the electrometer is not suitable for such experiments. The two experiments, however, confirm the results given by Mizuno in the paper already mentioned. Up to the optimum capacity the difference of potential in the secondary appears to be approximately a linear function of the capacity around the break. That is we can write

$$V_m = cK_1. \tag{41}$$

For capacities greater than the optimum capacity it follows, approximately, the law, already given in § 2,

$$V_m = I_0 \sqrt{\frac{L_2}{K_1}}. \tag{42}$$

[1] Phil. Mag., Vol. 1, pp. 147-159, 1901.

Fig. 19 represents the relation of V_m to K_1 calculated from the last equation for three different values of I_0, putting $L_2 = 1$ in each case.

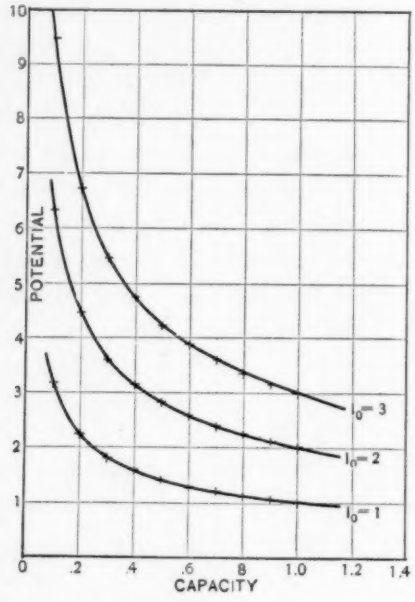

Fig. 19.

It is evident that the point where the straight line representing equation (41) and the curve representing equation (42) intersect will give the optimum capacity. This is shown in the curves given by Mizuno.[1] It is possible that the rounding off of his curves is due to the fact that he used a mercury break.

Johnson[2] has calculated the values of V_m from formula (42) for the cases observed by Mizuno and finds that the calculated and observed values show a close agreement.

As we have already stated, *after* the optimum capacity has been reached, the theory of the secondary potential is given approximately by (42). But no theory has as yet been given for the relation between the secondary potential, the primary capacity and the velocity of breaking *before* the optimum capacity is reached.

[1] Phil. Mag., Vol. 45, pp. 450 and 452, 1898.
[2] Phil. Mag., Vol. 49, pp. 216–220, 1900.

It is evident that a complete theory must take into account the temporary current across the break, the so-called "leakage current." In this case we shall have a divided circuit with three branches (see Fig. 20); the first branch containing an inductance and resistance, L_1 and R_1; the second, a resistance, R' (the resistance across the break), and the third, a capacity, K_1.

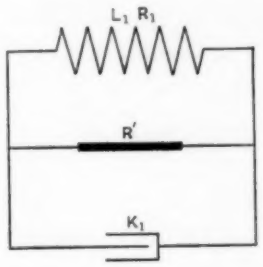

Fig. 20.

This problem has been investigated by Mizuno.[1] He, however, has not investigated our case, but the case in which, when $t = 0$, $I = 0$ and $q = Q_0$. In our case when $t = 0$, $I = I_0$ and $q = 0$.

In connection with this problem the question arises "How does the resistance increase as the distance between the break contacts increases?" This is a difficult question. Arons[2] has attempted to answer it by writing the resistance equal to

$$W\frac{\tau}{\tau - t}$$

where W is the resistance of the circuit before it is broken; t the time measured from the instant of breaking the circuit, and τ a short interval of time, measured from the instant of breaking, at the end of which the resistance becomes infinite. τ and W are, of course, constants and t is the variable.

Beattie[3] has carried on a series of experiments to determine the relation between the length of the secondary spark, the material of the primary poles, the suddenness with which they are separated, the capacity around the break and the magnitude of the electromotive force used to produce the primary current. His experiments are very interesting, but are qualitative and not quantitative. He used three speeds of breaking, which were only determined roughly as slow, moderately quick and very quick. The slow

[1] Wied. Ann., Bd. 4, pp. 811–814, 1901.
[2] Wied. Ann., Bd. 63, pp. 177–182, 1897.
[3] Phil. Mag., Vol. 50, pp. 139–148, 1900.

break was produced by separating the poles by hand; the moderately quick break had a velocity of about 800 cm. per sec., and the very quick break, a velocity of about 1,800 cm. per sec. It will be noticed that the quick velocities were much greater than those employed in this investigation.

He found that when the capacity of the condenser around the break was appreciable, the magnitude of the electromotive force in the primary made no difference so long as the current in the primary was kept the same by putting in non-inductive resistance: the length of the secondary spark was the same whether he used 10 or 100 volts to produce the primary current of 2.5 ampères. When, however, there was no condenser around the break, this did not hold, and the length of the secondary spark was found to depend very largely upon the primary electromotive force producing a given current. These results may be explained on the theory given above by supposing that when the capacity is very small the induced electromotive force in the primary is so great that the dielectric is completely disintegrated (possibly ionized) and forms an excellent conductor so that the greater the electromotive force of the primary, the greater will be the leakage current, and consequently the less the difference of potential in the secondary. When an appreciable amount of capacity is introduced into the primary the leakage current is very much reduced.

He experimented with platinum, copper, zinc and carbon poles. For platinum he found no difference in the results for the three different speeds; for copper only a small difference, and a large difference for carbon and zinc.

This dependence of the secondary spark length upon the speed of breaking in the case of copper, carbon and zinc is probably due to entirely different causes from those discussed above. The theory which has been developed is for two poles (such as platinum) which do not disintegrate under the electric stress, or under such heat as is developed, and it assumes a definite constant a which I have called the rupture coefficient. If the poles disintegrate the vaporized metal forms a conducting body between them, and a leakage current passes. The increase of the secondary spark length with the increase of the breaking speed for high speeds probably has

something to do with the time required to produce disintegration or vaporization of the poles.

In conclusion, I should like to express my thanks to Professor A. G. Webster for his unfailing kindness and helpfulness during the course of this research, and also to the authorities of Clark University for making it possible for me to carry it on.

I am indebted to Messrs. Morris E. Leeds & Co., of Philadelphia, for supplying me with materials with which to construct my coils, and for allowing me to wind them in their shops.

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CLARK UNIVERSITY, WORCESTER, MASS.,
June, 1901.

VISCOSITY OF LIQUIDS DETERMINED BY MEASUREMENT OF CAPILLARY WAVES.

BY FLOYD R. WATSON.

THE work about to be described grew naturally out of an experiment,[1] performed by the writer, in which surface tension of liquids was determined by measurement of capillary ripples. The ripple waves on a liquid surface were observed to decrease in amplitude as they progressed, apparently because of viscosity; hence the inquiry arose as to the possibility of finding the magnitude of this viscous force in terms of the quantities which could be measured. A theoretical investigation showed that such a relation existed, so that measurements were taken and viscosities calculated for a number of liquids.

Circular waves were used instead of plane parallel waves as in the previous work, because the crests were so much more sharply defined, leading to greater accuracy in measurement. The consequent theoretical discussion was more complicated so that approximations were necessary, yet the greater experimental accuracy was thought sufficient to offset the theoretical disadvantages.

The method of procedure in the development of the formula was to set up a function which satisfied the conditions of the experiment as if no viscous force were present, and then find the influence of viscosity by equating the time rate of decay of the energy of the waves to the dissipation. It was necessary first, then, to investigate the movement of circular waves on the surface of an inviscid, incompressible liquid subject to the condition that at the boundaries the normal velocity is zero. The analytical conditions to be satisfied are as follows:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0, \tag{1}$$

[1] PHYS. REV., Vol. XII., p. 257.

$$\frac{\partial \varphi}{\partial z} = 0 \quad \text{for} \quad z = -h, \tag{2}$$

$$\begin{cases} x = r \cos \theta, \\ y = r \sin \theta, \end{cases} \tag{3}$$

$$\varphi \propto e^{i\sigma t}. \tag{4}$$

Condition (2) is satisfied if

$$\varphi \propto \cosh k(z + h),$$

where h is the depth of the liquid used.

$$\therefore \frac{\partial^2 \varphi}{\partial z^2} = k^2 \varphi.$$

Equation (1) becomes by substitution of (3) and (2)

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} + k^2 \varphi = 0,$$

where the term

$$\frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}$$

is zero because of symmetry about the origin, so that the equation may be written

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + k^2 \varphi = 0. \tag{5}$$

The solution of this equation, having regard also for condition (4), is

$$\varphi = C s e^{i\sigma t} \cosh k(z + h), \tag{6}$$

C being a constant and s a function of r. Substituting in (5) we get

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + k^2 s = 0 \tag{7}$$

which is an equation of Bessel's Functions. The solution[1] of (7) is

$$s = A J_0(kr) + B Y_0(kr). \tag{8}$$

[1] Gray and Matthews, "Treatise on Bessel's Functions," Chap. VI., equation 104, where $n = 0$.

The Y_0 function is infinite when r is zero, but the value of $r=0$ is not used in the experiment, hence the function is retained for symmetry.

If kr is large we may write [1]

$$(9)\begin{cases} J_0(kr) = \sqrt{\frac{2}{\pi kr}}\left[\cos\left(\frac{\pi}{4}-kr\right)+\frac{1}{8kr}\sin\left(\frac{\pi}{4}-kr\right)\right] \\ Y_0(kr) = 0.116J_0 - \sqrt{\frac{\pi}{8kr}}\left[\sin\left(\frac{\pi}{4}-kr\right)-\frac{1}{8kr}\cos\left(\frac{\pi}{4}-kr\right)\right] \end{cases}$$

$$AJ_0(kr)+BY_0(kr) = (A+0.116)\sqrt{\frac{2}{\pi kr}}\left[\cos\left(\frac{\pi}{4}-kr\right)\right.$$

$$\left.+\frac{1}{8kr}\sin\left(\frac{\pi}{4}-kr\right)\right]-B\frac{\pi}{2}\sqrt{\frac{2}{\pi kr}}\left[\sin\left(\frac{\pi}{4}-kr\right)\right.$$

$$\left.-\frac{1}{8kr}\cos\left(\frac{\pi}{4}-kr\right)\right].$$

As A and B are arbitrary constants we may put (for a particular solution)

$$(A+0.116) = B\frac{\pi}{2} = C,$$

so that we get

$$AJ_0(kr)+BY_0(kr) = C\sqrt{\frac{2}{\pi kr}}\left[\left(1+\frac{1}{8kr}\right)\cos\left(\frac{\pi}{4}-kr\right)\right.$$

$$\left.+\left(\frac{1}{8kr}-1\right)\sin\left(\frac{\pi}{4}-kr\right)\right]$$

$$= C\sqrt{\frac{2}{\pi kr}}\left[\cos\left(\frac{\pi}{4}-kr\right)-\sin\left(\frac{\pi}{4}-kr\right)\right],$$

neglecting $\frac{1}{8kr}$ compared with unity. To justify this approximation consider the value of $kr=\frac{2\pi}{\lambda}r$. The smallest value of r used in

[1] Gray and Matthews, page 40, where, $n=0$.

calculation is 2.5 cm. and the largest λ is 0.4 cm.; therefore the greatest value of $\frac{1}{8kr}$ is $\frac{1}{300}$ approximately.

We have further

$$s = AJ_0(kr) + BY_0(kr) = C' \sqrt{\frac{2}{\pi kr}} \cos(kr + \varepsilon).$$

where ε is a phase angle, thus restoring the generality of eq. (9), or we may write

$$s = C'' \sqrt{\frac{2}{\pi kr}} e^{i(kr+\theta)},$$

where the imaginary part is to be thrown away. Substituting in (6) we have

$$\varphi = \frac{C_1}{\sqrt{kr}} e^{i(kr+\sigma t+\theta)} \cosh k(z + h). \tag{10}$$

To find the influence of viscosity it is necessary to determine the dissipation over a small area of the surface, and equate this value to the time rate of decay of the energy over the same area.[1]

Thus taking the dissipation over the area $r\theta dr$ and summing from r to $r + \lambda$ we have

$$2F = -\mu \int_r^{r+\lambda} \frac{\partial q^2}{\partial z} r\theta \cdot dr,$$

$$q^2 = \left(\frac{\partial \varphi}{\partial r}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2,$$

$$\frac{\partial q^2}{\partial z} = \frac{2A^2k^2}{r} \sinh k(z + h) \cosh k(z + h),$$

where

$$\varphi = \frac{A}{\sqrt{kr}} \cosh k(z + h) \sin(kr - \sigma t), \tag{11}$$

and the same degree of approximation is used as before.

[1] Lamb, "Hydrodynamics," section 301.

At the surface where $z = 0$ we obtain

$$2F = -2\mu A^2 k^2 \theta A \sinh kh \cosh kh.$$

To find the kinetic energy we have the expression

$$T = \frac{\delta}{2} \int_r^{r+\lambda} \varphi \cdot \frac{\partial \varphi}{\partial z} r \cdot \theta \cdot dr \qquad [z = 0;\ \delta = \text{density}]$$

$$= \frac{\delta}{2} A^2 \sinh kh \cdot \cosh kh \int_r^{r+\lambda} \sin^2 (kr - \sigma t)\theta \cdot dr$$

$$= \frac{\delta}{4} A^2 \theta \cdot \lambda \sinh kh \cosh kh.$$

The total energy is twice T, hence we get

$$\frac{d}{dt}\left(\frac{\delta}{2} A^2 \theta \lambda \sinh kh \cosh kh\right) = -2\mu A^2 k^2 \theta A \sinh kh \cosh kh,$$

or

$$\frac{d(A^2)}{A^2} = -4\frac{\mu}{\delta} k^2 dt,$$

$$A = A_0 e^{-2\nu k^2 t} \qquad \left[\nu = \frac{\mu}{\delta}\right]$$

$$= A_0 e^{-\frac{2\nu k^2}{v} r} \qquad \left[t = \frac{r}{v}\right],$$

which expresses the fact that the logarithm of the amplitude decreases uniformly as r increases. Substituting in (11)

$$(12) \qquad \varphi = \frac{A_0 e^{-\beta r}}{\sqrt{kr}} \cdot \cosh k(z + h) \sin (kr - \sigma t) \qquad \left[\beta = \frac{2\nu k^2}{v}\right].$$

The final step in the development is to get a relation between ν, ρ and r, where r is the distance from the origin of disturbance and 2ρ is the radius of curvature of the wave crest.

We have the expression

$$(13) \qquad \rho = \frac{1}{\frac{\partial^2 z}{\partial r^2}},$$

if z is the height of the wave and is small.

Now z is found in terms of r and ν by means of the relation

$$\frac{\partial \varphi}{\partial z} = \frac{\partial z}{\partial t} \quad \text{for} \quad z = 0, \tag{14}$$

that is

$$\frac{\partial z}{\partial t} = \frac{kA_0 e^{-\beta r}}{\sqrt{kr}} \sinh kh \cosh kh \sin (kr - \sigma t),$$

whence

$$z = \frac{kA_0 e^{-\beta r}}{\sigma\sqrt{kr}} \sinh kh \cosh kh \cos (kr - \sigma t) \tag{15}$$

$$= C \frac{e^{-\beta r}}{\sqrt{kr}} \cdot \cos (kr - \sigma t),$$

where C is written for $\frac{kA_0}{\sigma} \sinh kh \cosh kh$. Therefore

$$\frac{\partial^2 z}{\partial r^2} = C \frac{e^{-\beta r}}{\sqrt{kr}} [(\beta^2 - k^2) \cos (kr - \sigma t) + 2k\beta \sin (kr - \sigma t)]$$

using the same approximation as before.

This equation becomes simplified when we consider that all measurements of the waves were taken at the crests where z is a maximum; that is, from equation (15) where $\cos (kr - \sigma t) = 1$. It follows then that $\sin (kr - \sigma t) = 0$. The equation may then be written

$$\rho = \frac{1}{\frac{\partial^2 z}{\partial r^2}} = \frac{\sqrt{kr} \cdot e^{+\beta r}}{C(\beta^2 - k^2)}. \tag{16}$$

If the ratio is taken of two radii of curvature of waves n wave-lengths apart we get

$$\frac{\rho_n}{\rho_0} = \sqrt{\frac{r_n}{r_0}}\, e^{\beta(r_n - r_0)} \tag{17}$$

but $(r_n - r_0) = n\lambda$, and $\beta = \frac{2\nu k^2}{v}$, so that

$$\beta (r_n - r_0) = 2\nu k^2 n \frac{\lambda}{v} = 2\nu k^2 n T,$$

where T is the period of the waves. Substituting this value in (17) and taking logarithms to the base 10 we finally obtain

$$\nu = \frac{\mu}{\delta} = \log\left[\frac{\rho_n}{\rho_0}\sqrt{\frac{r_0}{r_n}}\right]\frac{1}{2k^2nT\,0.43429}, \tag{18}$$

which gives

$$\mu = f(\delta, \rho, r, \lambda, T),$$

where

μ is the coefficient of viscosity,
δ the density of the liquid,
r the distance of wave from origin of disturbance,
λ the wave-length,
T the period of the waves.

FORMULA FOR SURFACE TENSION.

The quantities which enter into the viscosity formula are sufficient also for a determination of the surface tension. To deduce a formula suitable for this case, the method used by Rayleigh[1] is followed.

In addition to the conditions (1), (2), (3) and (4) already discussed, there is yet to be satisfied the pressure equation

$$\frac{p}{\rho} = -gz - \frac{\partial\varphi}{\partial t}, \tag{19}$$

z being measured positively upwards.

At the surface the pressure due to capillarity is

$$p = -T\frac{\partial^2 z}{\partial r^2}, \tag{20}$$

if z denotes the height of the wave at the point r and T is the surface tension. Substituting this value in (19) we get

$$\frac{T}{\rho}\frac{\partial^2 z}{\partial r^2} = gz + \frac{\partial\varphi}{\partial t}. \tag{21}$$

Differentiating with respect to t and remembering that

$$\frac{\partial\varphi}{\partial z} = \frac{\partial z}{\partial t} \quad \text{for} \quad z = 0,$$

we obtain

$$\frac{T}{\rho}\frac{\partial^3\varphi}{\partial z\cdot\partial r^2} = g\frac{\partial\varphi}{\partial z} + \frac{\partial^2\varphi}{\partial t^2}. \tag{22}$$

[1] "Theory of Sound," Vol. II., Sec. 353.

The velocity potential suitable to the problem in hand is that expressed in equation (6) where

$$S = \frac{C_1}{\sqrt{kr}}\left[A_1 \cos\left(kr - \frac{\pi}{4}\right) - B_1 \sin\left(kr - \frac{\pi}{4}\right)\right]$$

giving the expression

$$(23)\quad \varphi = C_2 e^{i\sigma t} \cosh k(z+h) \frac{1}{\sqrt{kr}}\left[A_1 \cos\left(kr - \frac{\pi}{4}\right) - B_1 \sin\left(kr - \frac{\pi}{4}\right)\right].$$

Substituting in equation (22), we have

$$\frac{Tk^3}{\rho} = gk - \sigma^2 \coth kh.$$

The value of kh is never less than 15, so that coth kh may be replaced by unity and finally

$$(24)\qquad T = \rho\left(\frac{n^2\lambda^3}{2\pi} - \frac{g\lambda^2}{4\pi^2}\right).$$

where

ρ is the density of the liquid,
n the vibration frequency,
λ the wave-length,
g the acceleration of gravity.

It is to be noted that this formula is the same as the one deduced for plane, parallel waves,[1] thus indicating, to the degree of approximation used, that the two kind of waves are similar as regards the surface tension.

Description of Apparatus.

The apparatus used was that employed by the writer in determining surface tension of liquids.[2] A brief description, however, is given here to explain the modifications made for this experiment. Circular ripple waves were generated on a liquid surface by means of a small glass tube, which dipped into the liquid while its upper end was clamped to one prong of a vibrating tuning fork; its

[1] Phys. Rev., Vol. XII., p. 257.
[2] Phys. Rev., Vol. XII., p. 259.

motion thus being in a vertical line. The waves created were so small in amplitude as not to be seen by the unaided eye, so a stroboscopic method[1] was employed to make them visible. Interrupted light, isoperiodic with the waves, was allowed to fall on the liquid, thus causing the waves to appear as a series of light and dark bands. The light was made parallel before impingement on the liquid by reflection from a concave mirror, so that the waves could be conveniently viewed through a telescope which was firmly clamped on a dividing engine, about 70 cm. above the liquid.

Thus to measure the wave-length it was only necessary to focus on a wave crest, then, by means of the dividing engine, to move the telescope parallel to its first position until a number of waves had been passed over; the positions of the first and last waves being noted. The distance traveled by the telescope divided by the number of waves gave the wave-length.

To understand the method of measuring the radius of curvature, it is necessary to consider the reflection of light from a wave crest.

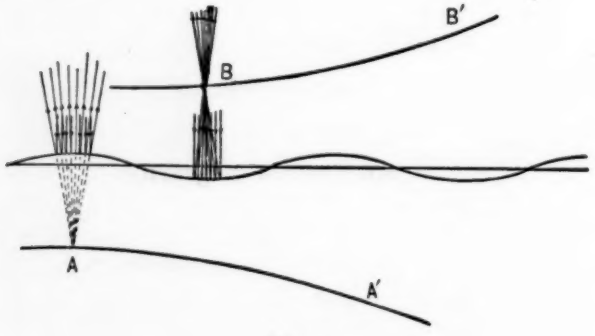


Fig. 1.

The figure represents a cross-section of the surface of the liquid. Each crest acts as a cylindrical convex mirror and causes the light to appear to come from the points A, A'. The distance from the crest to A is equal to one half the radius of curvature of the crest; as only a narrow band on the top of the crest is considered. To measure this distance, the telescope was first focused on A and the distance of A from the telescope read off—the telescope tube having

[1] Phil. Mag. (5), 30, p. 386.

been previously calibrated for this purpose. The telescope was next focused on the liquid surface and its distance from the telescope ascertained. The difference between these two measurements gave one half the radius of curvature.

The vibration frequency, n, of the waves was the same as that of the fork which generated the waves, so that n could be found by getting the period of the fork. This was done by a stroboscopic method. A slotted disc was rotated until its speed was such as to make the fork appear to stand still when viewed through the slits of the disc. The speed of the disc was carefully determined and n calculated.[1]

This process, however, was too elaborate for each measurement, so a second fork was tuned in unison with the first and set aside for comparison. Any subsequent variation in the period of the first fork could then be easily detected and adjustment for unison made, if necessary. In this way the vibration frequency was kept constant. Once during the experiment the standard fork changed its period due to a weight dropping off, so a redetermination of the period was made by the slotted disc method.

Method of Taking Measurements.

All apparatus to come in contact with the liquid was thoroughly cleaned.[2] The tuning forks were started and compared with the standard fork. The glass pan was then filled with liquid to a depth of a centimeter or more, and the tube vibrator adjusted so that it dipped just below the liquid surface and was perpendicular to it. The liquid itself was drawn by a siphon from the interior of the supply and not poured from the surface—thus avoiding possible surface impurities. The apparatus was arranged symmetrically; that is, so that the rays of light were parallel to a plane which was perpendicular to the liquid surface, and which passed through the axes of the telescope and mirror. Then the telescope was focused on the outer rim of the vibrating tube where it cut the liquid and was moved out along the direction of propagation of waves about 3 cm. and focused on a wave crest. The distance traversed, r,

[1] Phys. Rev., Vol. XII., p. 266.
[2] Phys. Rev., Vol. XII., p. 264.

was noted, and the radius of curvature, 2ρ, measured several times. The telescope was moved again over five more waves and focused on the fifth crest, the distance traveled being noted and 2ρ determined as before. This process was continued until 25 or 30 waves were passed over. The wave-length was next determined in the way already described. The distance from the telescope to the liquid was found by focusing the telescope on a bit of paper floating on the surface. Finally the temperature and density of the liquid were ascertained, thus completing the measurement of all the quantities necessary to a determination of the viscosity and surface tension.

LIQUIDS USED.

As cleanliness was an essential feature of the experiment, great care was taken to have the liquids clean. Thus the mercury was purified[1] by passing it through strong H_2SO_4 and allowing it to remain in contact with pasty mercurous sulphate. The petroleum and turpentine used were the best commercial articles. Water measurements were taken on tap water, as this was found as satisfactory as distilled water[2] and more convenient.

Other liquids used were from Merck & Co., Darmstadt, and were guaranteed to be pure.

GRAPHICAL TEST OF MEASUREMENTS.

Before the actual calculation of the viscosity was carried out, the measurements were subjected to a graphical test. Thus if the distances from the vibrator were plotted as ordinates and the logarithms of the corresponding ρ's plotted as abscissæ, the resulting points were found to lie nearly on a straight line, if the measurements were regular. This test was discovered in the early stages of the experiment before a rigorous proof of the viscosity formula had been made, hence it was of especial value then as a check on the correctness of the measurements. The clue leading to this test is as follows,—if the cross section of a wave crest is assumed to be

[1] PHYS. REV., Vol. XII., p. 264.

[2] See T. P. Hall, Phil. Mag. (5), 36, 385; Rayleigh, Phil. Mag. (5), 33, 363.

part of a circle (see Fig. 2), then it may be shown geometrically that

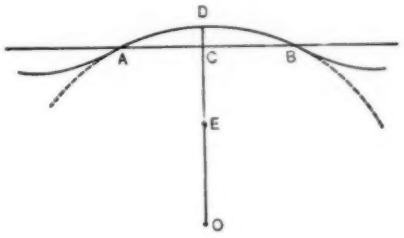


Fig. 2.

$$\overline{DC}^2 - 4\overline{DE}\cdot\overline{DC} + \overline{AC}^2 = 0,$$

or

$$\theta^2 - 4\rho\theta + \left(\frac{\lambda}{4}\right)^2 = 0, \qquad \text{(I.)}$$

where

θ is the amplitude of the wave,

2ρ is the radius of curvature,

λ is the wave-length.

Neglecting θ^2 in comparison with θ, (I.) becomes

$$\rho = \frac{\lambda^2}{64}\cdot\frac{1}{\theta}. \qquad \text{(II.)}$$

θ may be written

$$\theta = A\frac{e^{-\beta r}}{\sqrt{r}},$$

where the term $e^{-\beta r}$ is due to viscosity and $\sqrt{r}$ to the spreading of the waves in circles, so that (II.) is now

$$\rho = Ce^{\beta r}\cdot\sqrt{r}. \qquad \text{(III.)}$$

Taking logarithms

$$\log\rho = \beta r + \log\sqrt{r} + \text{constant}, \qquad \text{(IV.)}$$

which gives a straight line. In plotting results $\log\sqrt{r}$ was found small in comparison with βr and therefore neglected.

This equation is easily obtained also from the general theory. Equation (16) may be written

$$\rho = \text{const}\, e^{+\frac{2\nu k^2 r}{v}} \sqrt{kr},$$

taking logarithms we get as before

$$\log \rho = \beta r + \log \sqrt{kr} + \log (\text{const}),$$

where

$$\beta = \frac{2\nu k^2}{v}.$$

Table I. and Fig. 3 give an idea of the measurements taken.

TABLE I.

Liquid.	Distances from Vibrator in cm.						Values for ρ in cm.					
	r_1	r_2	r_3	r_4	r_5	r_6	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
I. Water.	3.111	5.070	7.048	8.999	10.943	12.864	10	15.7	26.5	39.7	59.7	78.0
II. Mercury.	3.22	4.627	6.033	7.44	8.84	10.252	11.6	18.1	26.8	36.1	55.6	73.6
III. Petroleum ether.	2.229	3.666	5.101	6.525	7.967	9.437	7.9	10.6	13.0	17.6	23.1	32.1
IV. Amylacetate.	2.396	3.844	5.260	6.671	8.087		4.9	10	16.4	29.4	53.3	

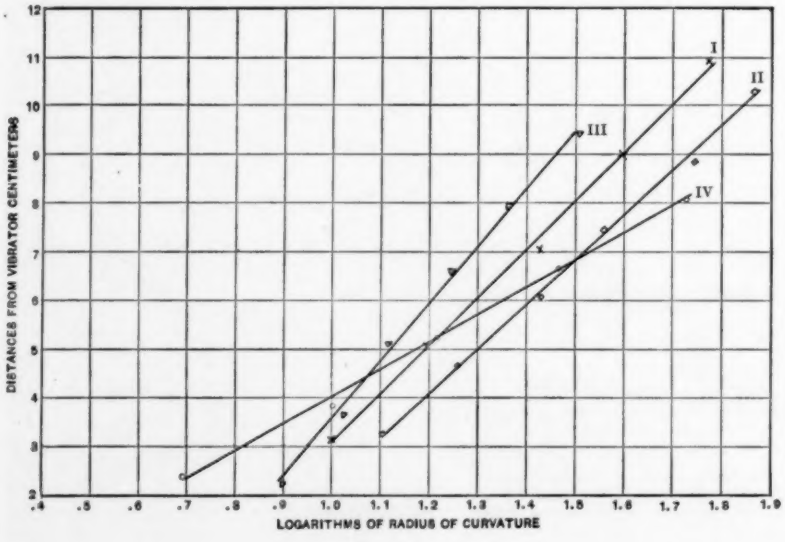


Fig. 3.

If this preliminary test showed no great variation in measurements, calculations could be made according to equations (18) and (24), giving numerical values for the viscosity and surface tension.

Amplitude of Waves.

Before writing down the table of results obtained it will be of interest to give an idea of the extreme smallness of the amplitude of the waves. By means of equation (II.)

$$\theta = \frac{\lambda^2}{64}\frac{1}{\rho}$$

the following maximum and minimum values were obtained (see Table II).

Table II.

Liquid.	r	λ	ρ	θ
Water.	14.0	0.3876	160.0	0.00001465
"	0.917	0.3939	3.4	0.000712

r is the distance of wave crest from vibrator.
λ, the wave-length.
ρ, one-half the radius of curvature.
θ, the amplitude.

All the measurements were reduced to centimeters.

These values were so small as to seem almost incredible, so a verification of some kind was sought. No direct method of measuring the amplitude was thought of, but a reasonable estimate was found by measuring the amplitude of vibration of the tube which generated the waves. The method used was as follows. The apparatus was set up and measurements taken for r, ρ and λ as close to the vibrator as possible. Then, without disturbing the fork, the vertical movement of the vibrating tube was observed by means of interrupted light and a microscope. The microscope was firmly mounted and could be raised or lowered by means of a micrometer screw. The interrupted light was nearly in period with the fork, so that the tube appeared to move slowly up and down. The positions of extreme swing were noted. The distance between these

positions divided by two gave the amplitude. The results are shown in Table 3:

TABLE III.

Liquid.	r	λ	ρ	θ
Water.	.917	.3939	3.4	0.000712
"	1.726	"	4.1	0.000589
"	2.526	"	5.0	0.000484
"	2.338	"	6.1	0.000396
Vibrator.				0.00379

From this it is seen that the wave 0.9 cm. from the vibrator has approximately one-fifth the amplitude of the tube vibrator. The conclusion seems to be that the liquid slips on the tube, or else the motion rapidly diminishes near the vibrator. The first conclusion seems the better one as shown by Fig. 4:

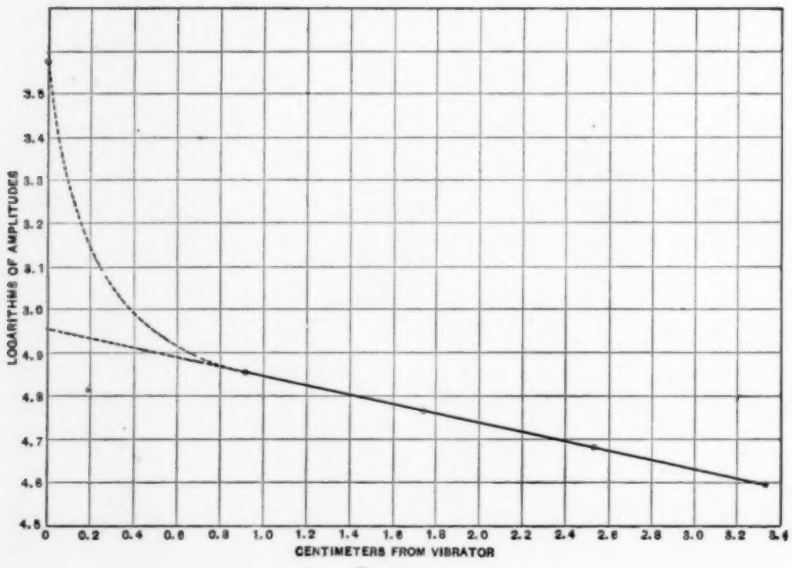


Fig. 4.

The logarithms of the ρ's are plotted as ordinates and the r's as abscissæ. If the straight line thus formed be extended to cut the position of the vibrator, the value of the amplitude to correspond is much less than the value actually observed; therefore, it seems probable that the liquid slips on the vibrator.

TABLE IV.

Liquid.	δ	n	λ	t	Viscosity.	Tension.	t	Viscosity.	Observer.	t	Tension.	Observer.
Acetic ether.	0.899	88.488	0.2762	17°C.	0.00437	22.1	20	0.00561	Graham	10.4	25.52	Mendelejen.
" "	"	"	0.2781	"	0.00427	23.0	20	0.00460	Gartenmeister.	24.6	25.64	Wilhelmy.
Amyl acetate.	0.862	88.65	0.2838	25.6	0.00736	24.55						
Amyl alcohol.	0.812	88.488	0.2800	21.8	0.02290	20.7	10	0.06234	Pagliani.	15	24.45	Mendelejen.
							20	0.03696	Graham.			
Benzol.	0.875	"	0.2968	18.5	0.00434	26.9	12	0.00739	Wijkander.			
"	"	"	0.2991	"	0.00411	27.3	20	0.00645	"	15	28.77	Schiff.
							20	0.00645	Gartenmeister.			
							12	0.00617	Wijkander.	12.5	28.12	Bede.
Chloroform.	1.489	"	0.2580	12.3	0.00525	29.6	20	0.00568	"	20	27.24	Magie.
							20	0.00568	Gartenmeister.			
Mercury.	13.6	88.65	0.2880	19.7	0.0629	378.0	0	0.01697	S. Koch.		502.8	Worthington.
"	"	"	0.2888	"	0.0702	379.0	10.1	0.01631	"		515.0	G. Meyer.
"	"	"	0.2878	20	0.0577	376.0	16.7	0.01592	"			
"	"	"	0.2875	"	0.0626	380.0	17	0.01602	Warburg.			
"	"	"	0.2875	"	0.0628	380.0	17.1	0.01543	Th. Schmidt.			
"	"	"	0.2892	21.8	0.0614	384.0						
Petroleum.	0.795	"	0.2965	18.4	0.0114	25.51				18	24.41	Magie.
Petroleum ether	0.635	"	0.2913	11.0	0.0021	19.6						
Turpentine.	0.859	"	0.2936	20.7	0.0102	25.1						
"	"	"	0.2963	"	0.0106	25.0						
"	"	"	0.2968	21	0.0107	25.0						
"	"	"	0.2907	21.5	0.0101	24.5						
Water.	1	"	0.3954	5.3	0.0131	73.4	0	0.01775	Pagliani.	18	73	Hall.
"	"	"	0.3992	5.8	0.0111	75.8	10	0.01309	"	18	73.17	Dorsey.
"	"	"	0.3926	10.3	0.0117	72.6	15	0.01124	Sachs.			
"	"	"	0.3950	10.5	0.0110	73.2	17	0.01105	W. König.			
"	"	"	0.3936	12.5	0.0098	72.4	17	0.01106	Grotian.			
"	"	"	0.3900	15.1	0.0096	70.4	17.5	0.01050	Petroff.			
"	"	"	0.3905	15.1	0.0086	70.5	20	0.01032	Gartenmeister.			
							20	0.01020	Drew.			

RESULTS OF MEASUREMENTS.

Table IV. gives the results obtained. ∂ is the density of the liquid, n the vibration frequency of the waves, λ the wave-length in cm., t the temperature in centigrade scale and the *tension* the surface tension in dynes per cm. For purpose of comparison, the results of other observers are written down. The data is nearly all obtained from Landolt and Börnstein's tables.

DISCUSSION OF RESULTS.

An inspection of the table shows that (excepting mercury) the *surface* viscosity as determined by this method has practically the same value as that obtained by other methods for the viscosity of the *interior* of the liquid.

Results for both viscosity and surface tension are a little less than values obtained by other observers. At first it was thought that the discrepancy was due to the use of circular waves by introducing a variation in the wave-length. This supposition is not probable when the following measurements of the wave-length are considered (see Table V.):

TABLE V.

Turpentine.	Petroleum.	Mercury.
0.304	0.300	0.296
0.289	0.300	0.289
0.301	0.300	0.296
0.294	0.298	0.289
0.302	0.293	0.291
0.294	0.297	0.291
0.295	0.292	0.285
0.296	0.301	0.291
0.298	0.298	0.294
0.303	0.292	0.280
	0.303	0.300
	0.298	0.291
		0.291

The results give the wave-length (in cm.) of consecutive waves, the first one being at least 3 cm. from the vibrator. These values show the wave-length to be constant, within observational errors, in

the region were measurements were taken; hence the cause of the variation in viscosity must be sought elsewhere. It may be that the conditions of the liquid at the *surface* make the viscosity different from its value in the *interior*. Another possible cause of the variation may be found in the differences in method. Other methods involve a friction of the liquid with a solid, while in this method the viscosity is determined from observation of a free liquid surface. The most probable explanation is that the liquid surface becomes contaminated by the continued exposure to the air during the time measurements were taken, so that the surface may never be counted on as being free from dissolved gases or dust particles.

The two liquids which gave the most trouble were water and mercury. Contamination of the surface of either of these liquids was more noticeable than for other liquids used. Thus with mercury there is a drop in the tension and a rise in the viscosity. With water, the variation in viscosity as the temperature increases has not the regularity as shown by results of other methods; and even at the same temperature the results are not always concordant. In the case of a liquid like turpentine, however, the values are the same, within errors of observation, for the same temperature.

Mercury viscosity values are about four times as great as might be expected, so that on first thought it might be inferred that conditions at a mercury surface are different than at other liquid surfaces. This inference is seen to be incorrect if the values of the surface tension are considered. These values average about 380 dynes, which is the same value as found for the tension between mercury and *water*.[1] The conclusion is drawn at once that the mercury surface is contaminated by a thin film of water (*i. e.*, moisture), and that the viscosity determined must be that for a mercury-water surface instead of mercury-air. This tendency of a clean mercury surface to become contaminated is ever present,[2] so that it is improbable that mercury viscosity can be determined by this method.

Two attempts were made to get better results by cleaning the mercury surface with a glass slide. In the first trial the tension was raised from 382 dynes to 402, and the viscosity lowered from 0.080

[1] PHYS. REV., Vol. XII., p. 267.

[2] Phil. Mag. (5), 33, p. 363. Contamination of liquid films, by Lord Rayleigh.

to 0.073. In the second trial the tension was raised successively from 382 dynes to 403 and 420, while the viscosity *increased* from 0.074 to 0.076 and 0.080. Thus the behavior of a mercury surface is erratic, the contact of the air and consequent impurities modifying its action.

Other liquid films are not free from annoying features. Thus the surface of a volatile, non-homogeneous liquid undergoes slow distortions. This action is due to the unequal rates of evaporation of different parts of the surface, causing a local change in the surface tension and a consequent movement of the surface film. This effect was found with benzol, petroleum-ether and chloroform.

CONCLUSION.

The following observations seem important—*first*, that the method is best applicable to liquids of low surface tension; *second*, that the most favorable conditions for experimental success require a steady wave pattern, and the greatest possible cleanliness of liquids and containing vessels; *third*, that, in order to satisfy theoretical assumptions, the depth of the liquid should be one centimeter or more, and that all measurements of the waves be taken at least two centimeters from the vibrator.

In conclusion, the writer desires to express his thanks to Professor Nichols for suggestions and encouragement during the course of the experiment; also to Professor Merritt and Professor McMahon for assistance in the interpretation of the theory involved. The writer is further grateful to Dr. Chamot, of the Chemistry Department, who has been especially kind in offering suggestions and loaning chemicals.

LABORATORY OF CORNELL UNIVERSITY,
April 15, 1902.

WIRELESS TELEGRAPHY WITH A RELAY-TELEPHONE RECEIVER.

By A. H. Taylor.

THE relay-telephone receiver for wireless telegraphy consists essentially of a dry cell, a high resistance relay, and a telephone receiver. Hence in simplicity it excels most if not all devices which depend upon the use of the ordinary coherer. A study of coherers, their construction and uses, has led to the conclusion that a single-contact coherer, such as that described by Guthe and Trowbridge in "The Theory of the Coherer,"[1] would be the most sensitive to electrical displacements. But the difficulties to be overcome in the practical use of such a coherer are many. It must be automatically decohering, and not require continued adjustment.

A coherer may be used for the detection of electrical waves in two general ways. First, it may be adjusted until the initial resistance is infinite, and yet the particles are close enough together to be cohered by the wave. With a multi-contact coherer this condition is fairly easy to meet, and it is quite possible also with a single-contact coherer, although frequent adjustment is generally necessary, since all depends on one gap at the contact place. If this gap becomes a little too great, the instrument is no longer sensitive. Hence decoherence of a single-contact coherer, however it is accomplished, is liable to change the gap at the contact place and destroy the sensitiveness.

Second, a coherer may be adjusted so that its original resistance is high but finite. In this case a very slight change of the current through the coherer produces large fluctuations of the resistance. When in this condition, the instrument is by far more sensitive to electric waves than when used with the initial resistance infinite. This adjustment is not very difficult to obtain, but it is very difficult

[1] Physical Review, July, 1900.

to decohere without destroying the contact altogether. This is especially true of a single-contact coherer, which is liable to be located in a building more or less subject to jars and vibrations.

While experimenting with single-contact coherers, the secondary circuit of a relay was used, but with rather poor success. This coherer action of a relay contact was observed some years ago, but was not very thoroughly investigated. The practical difficulty with the relay was, as with other coherers, that decoherence was liable to destroy the adjustment, and the platinum contacts did not seem as sensitive to electrical waves as a nickel coherer.

So nickel contacts were substituted for the platinum, and the result was a very sensitive coherer. The contacts were then reversed, so that the relay *opened* circuit on the down stroke as an electric bell does. Then the relay, a sensitive telephone receiver, and a dry cell, were connected as shown in Fig. 1, the primary of the relay being in series with its secondary and the telephone. Fig. 2 shows the arrangement of the contact of the relay.

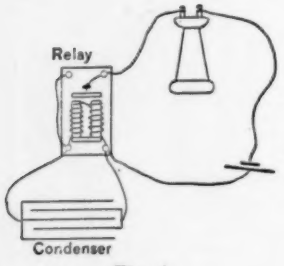


Fig. 1.

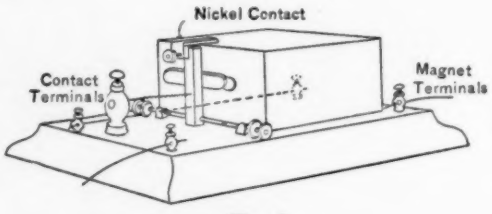


Fig. 2.

The condenser across the primary terminals of the relay makes the signals received sharper, for it counteracts the E.M.F. of self-induction of the relay, which would of itself cohere the secondary for fine adjustments.

The box relay used had a resistance of 179 ohms, and the telephone receiver had 100 ohms, while the apparent resistance of the contact varied from 350 to 30 ohms.

To adjust this receiver, the contacts were brought very close together, and the spring was tightened until the purring which first occurred in the telephone just ceased. A little practice made this adjustment easy and permanent. The whole receiver, excepting

the vertical wire and the ground connection, could be set up and adjusted in five minutes.

When a wave strikes this receiver there is heard in the telephone a rattle or purring noise, which ceases when the wave ceases. It is strong or faint, according to the distance from the sending apparatus.

It is so sensitive as to be affected by a spark from another relay secondary at a distance of one hundred feet. The sending apparatus used in nearly all of the tests to which this receiver was subjected, consisted of a four-inch induction coil run from a 100-volt A. C. circuit by means of the carbon interrupter described in the February number of this journal. To this coil were connected inductance and capacity in the proportion to get oscillations.

The inductance was made large enough to be well within the limit of the critical relations $R^2 < \frac{4L}{C}$, where R = resistance of spark gap and circuit, L the inductance, and C the capacity of the condenser, including the vertical wire. The period of the waves used as calculated from the approximate formula $T = 2\pi \sqrt{LC}$, was 0.9×10^{-6} seconds. A 35-foot vertical wire was used, and a ¼-inch spark between brass knobs one inch in diameter. One side of the coil was grounded.

A test made at a windmill one mile from the laboratory gave very satisfactory results with nine feet of vertical wire at the receiving end.

The sending and receiving circuits were not in synchrony during any of the tests made.

At a distance of four miles from the laboratory the receiver required thirty-two feet of vertical wire, while at any point within one-half mile, no vertical wire at all was required, but a good ground from one side of the relay secondary to the steam pipes of the building in which the receiver happened to be located was found necessary. Several trials at a distance of less than one-quarter of a mile with a ground on a hydrant or a long rod driven several feet into the earth gave very poor results unless a vertical wire was used. This led to the conclusion that the steam pipes in the buildings where the other tests were made acted as vertical wires, or at least carried oscillations which affected the relay coherer.

It was found unnecessary to ground directly on the steam pipes. One side of the relay secondary was connected to an *open* coil of a few turns of wire, insulated, and a radiator was connected to one end of another open coil surrounding the first. The receiver worked nearly as well as with the direct ground.

An ammeter reading to ten thousandths of an ampère, was put in series with the relay while signals were sent every second. The fluctuations of the needle showed negative coherer action to take place nearly as often as positive. Therefore the primary of the relay was cut out, and the coherer worked rather fainter, but very regularly and consistently, taking any number of signals without being touched. It was thought at first that the primary of the relay was necessary to produce decoherence. But while it does increase the effect and render the instrument more sensitive, it is not by any means a necessity. In other words, the single-contact coherer, if properly adjusted, is automatic, and depends equally upon the positive and negative cohering effects of successive waves. This curious behavior is perhaps made clearer by reference to the two following curves. These were obtained by the method used by Guthe and Trowbridge, with some slight modifications. R_1 is the relay,

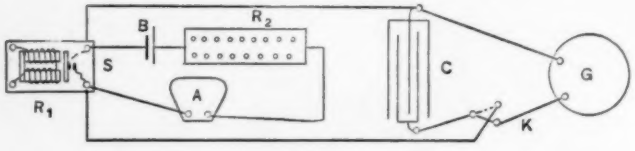


Fig. 3.

whose secondary S is in series with a 2-volt cell at B and a resistance box R_2. The condenser C is connected through the charge and discharge key K to the secondary terminals S and the galvanometer G. The ammeter at A reads to ten thousandths of an ampère, and by using a shunt reads up to 1.50 ampères. The series of observations for these curves extended over a range of current from 0.00005 to 1.50 ampères, but those of the series were chosen which covered the range within which the instrument is used in practice. From these curves we see that between the limits of 250 to 100 ohms a very slight change in current causes a very great change in

resistance. This suggests, but does not explain. A series of experiments are now being conducted in view of determining the relation between these curves, which are a fair sample of many others taken since, and those obtained by Guthe and Trowbridge. The writer has obtained many curves similar to their's but under very different conditions from those under which these curves were taken. It seems that the behavior of a coherer carrying a very small current at low E.M.F. is very different from the behavior of the same coherer *after coherence*, or when carrying larger currents.

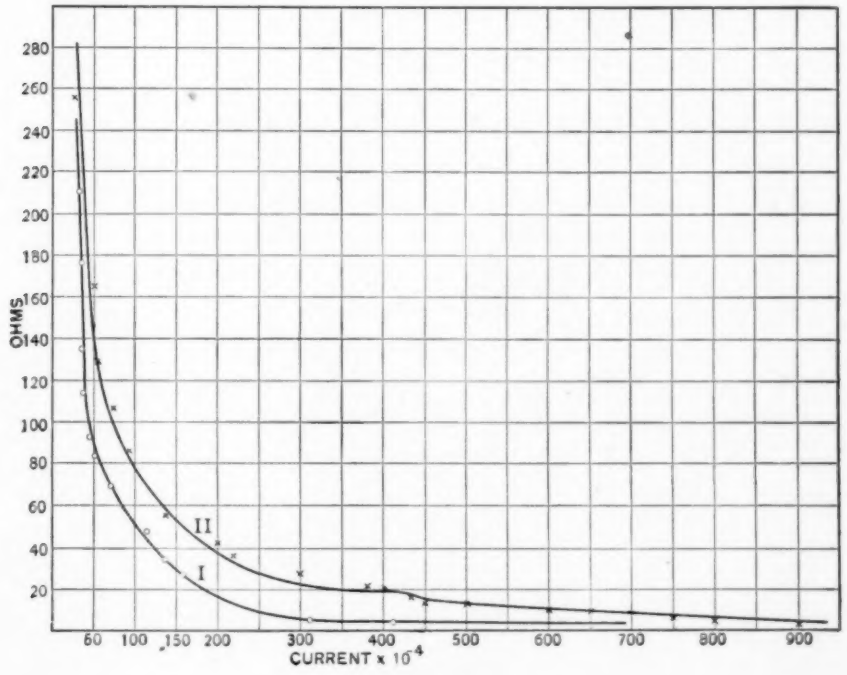


Fig. 4.

THE VOWEL I^e (AS IN PIT).

BY LOUIS BEVIER, JR.

THE vowel which is the subject of this study is in some respects a difficult one, and the results are not as consistent or as conclusive on some points as those obtained for the vowels described in previous articles. For this there is a good reason. It lies chiefly in the fact that the sound i^e in actual speech is pronounced uniformly short, and many speakers find difficulty in prolonging it. Then, again, it occurs in our usual American speech only in closed syllables. To isolate it and pronounce it by itself as an individual sound requires for some voices considerable practice.

Hence it is that it was found quite difficult to secure good singing records of i^e. Some persons were entirely unable to prolong the sound in singing without changing its quality, and some could not hold it uniformly true but would attach to it various glides. This becomes noticeable to the attentive ear when such records are reproduced, and is most conspicuous to the eye in the transcribed waves. Some specimens, instead of showing substantial uniformity throughout their whole extent, exhibited a continuous progressive change, *i. e.*, they were neither simple sounds, nor even diphthongs, but composites of numerous different resonances in rapid succession.

The analyses given in the table (pp. 46, 47) were carefully chosen from those specimens which sounded true to the ear as reproductions, and which remained reasonably uniform in wave character throughout their duration. Voice 1 produces a uniform i^e as readily as any other vowel, and voices 4 and 7 succeeded readily enough after a little drilling.

No such difficulty inheres in the study of i^e as spoken, but the spoken vowels present special problems and have been deferred for the present. The difficulty arises when the subject is asked to sing

*i*ᵉ at a registered pitch, and to prolong it beyond the habit of spoken utterance.

The following table (pp. 46, 47) of analyses will afford, I believe, a sufficient basis for a satisfactory acoustic definition:

Examining this table it will be found that there is a marked region of resonance centering at about 1850. If we begin for the records of lowest pitch and study the table record by record this will become entirely clear. The black faced figures will readily guide the eye in selecting the important upper partial in this region of resonance. Here there is an element which, while not absolutely a constant, is nearly enough so to be important as a differentiating factor.

It will be observed that this region of resonance is slightly higher than that found for *e*, but not enough higher to be a mark of differentiation (by itself) to the ear, or in the visible trace to the eye. So far as the pitch of this resonance region is concerned, the analyses of *e* and *i*ᵉ on the same fundamental are often in entire agreement. How complete this agreement is will be apparent if the table of *e* analyses published in the April number be laid side by side with the present table. In both on I. 152 XII. is the prominent upper partial, on I. 160 it is XI., on I. 181 X., on I. 256 VII. (for *i*ᵉ sometimes VIII.), on I. 384 V., etc. Now and then the important upper partial of *i*ᵉ will be higher pitched than that of *e*, so that when an average is to be struck we arrive at a frequency, characteristic for *i*ᵉ, higher by a small fraction of a tone than for *e*.

Over against this close similarity it is not hard to find as striking a contrast. As we examine the records at pitches below this region of resonance, we find that between 1,700 and 650 the normal *i*ᵉ has little resonance at any point. The upper partials lying between those frequencies are seldom present with an amplitude percentage over 4.0 or 5.0 (save on high fundamentals where we expect forced resonance) and the average is even lower. For *e* on the other hand we found a strong resonance region centering at about 1,050, and more or less resonance from that point upward until the upper resonance maximum was reached. This mid-resonance is wanting for the normal *i*ᵉ, and gives it quite a different resonance curve, and of course quite a different acoustic character.

ANALYSES OF THE I^E-CURVE.

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
		152	304	456	608	760	912	1064	1216	1368	1520	1672	1824	1976	2128	2280	2432	2584
1	68.8	45.3	8.1	**6.4**	**5.5**	2.9	1.5	1.5	3.2	1.5	2.6	3.5	**9.0**	3.2	0.3	0.0	2.3	3.2
		160	320	480	640	800	960	1120	1280	1440	1600	1760	1920	2080	2240	2400	2560	2720
6	50.8	28.2	8.8	**9.8**	**10.5**	2.4	1.0	2.4	1.0	5.4	2.0	**15.5**	5.4	3.2	1.6	1.2	1.6	0.0
		171	342	513	684	855	1026	1197	1368	1539	1710	1881	2052	2223	2394	2565	2736	2907
1	105.0	42.3	3.2	**7.6**	**6.1**	1.2	4.2	2.5	2.0	5.5	**12.3**	4.8	1.5	1.2	2.6	1.1	0.6	1.2
		181	362	543	724	905	1086	1267	1448	1629	1810	1991	2172	2353	2534	2715	2896	3077
1	88.8	26.7	9.0	**12.7**	3.5	2.4	1.1	2.7	3.6	4.7	**16.9**	3.3	1.6	5.5	3.2	0.2	1.8	1.6
		192	384	576	768	960	1152	1344	1536	1728	1920	2112	2304	2496	2688	2880	3072	3264
6	72.9	46.6	3.3	**6.3**	2.6	0.2	1.2	1.9	3.3	9.6	**14.5**	4.3	1.9	0.6	1.2	0.6	1.1	0.8
7	56.4	48.9	4.6	**5.0**	2.1	1.6	2.7	1.4	6.2	5.5	**12.2**	1.4	2.3	2.3	1.9	1.6	0.4	0.2
		202	404	606	808	1010	1212	1414	1616	1818	2020	2222	2424	2626	2828	3030	3232	3434
1	73.1	21.4	**14.0**	**10.0**	0.6	5.8	1.2	4.1	10.0	**20.7**	2.2	2.6	2.2	1.0	0.6	1.3	0.8	1.5
		226	452	678	904	1130	1356	1582	1808	2034	2260	2486	2712	2938	3164	3390	3616	3842
1	167.4	33.3	**14.8**	**12.1**	0.8	0.8	5.5	**10.3**	4.7	3.0	2.3	3.1	3.4	0.8	2.1	2.1	0.8	0.1

ANALYSES OF THE I^E-CURVE.—*Continued.*

		256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	4352
1	281.6	32.8	**20.5**	4.1	1.9	3.4	12.4	**10.7**	1.6	5.8	3.8	3.0						
1	290.5	24.8	**13.0**	3.1	0.8	1.1	2.5	9.0	**24.8**	6.1	3.2	3.0	1.3	1.8	1.2	1.5	1.7	1.1
6	111.1	21.5	**15.3**	3.8	5.2	1.8	8.9	**17.6**	**11.4**	1.6	7.2	2.1	1.1	0.0	0.9	0.6	0.3	0.7
4	95.2	20.5	**14.4**	7.6	7.6	3.0	6.6	4.6	**15.5**	4.0	2.5	2.6	3.8	0.8	2.2	0.5	1.6	1.2
5	58.0	28.0	**10.0**	4.2	4.5	1.0	8.6	9.7	**13.1**	3.1	4.5	2.8	2.1	1.4	2.0	1.3	2.7	1.0
7	95.7	23.0	**10.5**	2.9	7.6	2.7	5.4	8.5	**28.8**	1.9	0.6	2.5	2.0	0.6	0.3	0.6	1.4	0.7
		272	544	816	1088	1360	1632	1904	2176	2448	2720	2992						
1	289.4	24.7	**23.9**	2.8	3.7	7.9	**18.0**	**12.5**	3.4	0.6	0.4	1.9						
		320	640	960	1280	1600	1920	2240	2560	2880	3200	3520	3840	4160	4480	4800	5120	5440
5	87.0	7.9	**37.1**	6.3	12.8	7.0	**16.4**	1.2	2.8	1.6	2.6	0.9	0.6	0.6	0.7	0.6	0.5	0.4
7	147.2	9.6	**26.9**	0.2	8.5	15.9	**27.5**	1.0	3.7	2.2	0.8	0.5	1.2	0.3	0.9	0.2	0.4	0.2
		384	768	1152	1536	1920	2304	2688	3072									
5	145.0	8.2	**27.1**	10.5	15.1	**24.1**	3.7	6.8	4.5									
7	121.8	14.1	9.6	7.2	20.5	**38.5**	2.4	3.2	4.5									
		512	1024	1536	2048	2560	3072	3584	4096									
4	150.4	36.3	32.4	10.1	**12.1**	5.5	2.6	0.5	0.5									
7	317.9	19.4	20.8	25.8	**29.3**	2.0	0.7	1.0	1.0									
		640	1280	1920	2560	3200	3840	4480	5120									
10	215.8	68.2	9.2	**16.8**	3.4	0.8	0.7	0.5	0.4									

When we descend the musical scale still further we find for *i*ᵉ a low region of resonance at about 575. This corresponds quite closely with the lowest resonance of *a*ᵉ and *ɛ* centering respectively in 650 and 620. For *i*ᵉ the point of maximum resonance here seems to be about a semitone lower than that of *ɛ*, and a little less than a whole tone lower than that of *a*ᵉ. This resonance region is obvious from the table. On I. 152, and I. 160 it is distributed between III. and IV., on I. 171, I. 181, I. 192, I. 202, I. 226 it falls on III., on I. 256, I. 272, I. 320 it reinforces II., and above that pitch is not clearly to be detached from the fundamental.

A final differentiating mark for *i*ᵉ is to be seen in the comparatively large amplitude of the fundamental or chord-tone itself. A rough computation of averages gives the following amplitude percentages of the chord-tone, for *a* 12.0, for *a*ᵉ 14.1, for *ɛ* 14.3, and for *i*ᵉ 28.7, while then *a*, *a*ᵉ, and *ɛ* give on an average but a small amplitude to the chord-tone, for *i*ᵉ more than 25 per cent. of the entire amplitude is to be found there. This accords well with the fact that *i*ᵉ is a "close" vowel. The tube through which the vowel is uttered is a narrow one without wide resonance chambers anywhere to give full and free reinforcement to the upper partials.

It must, however, be borne in mind that the proportions are considerably changed when we compute in terms of energy instead of simple amplitudes. By no means 25 per cent. of the energy of *i*ᵉ is in the chord-tone. It is not necessary to make these computations for the difference between *ɛ* and *i*ᵉ, though less striking, would be none the less real.

Looking at the chord-tone in the individual records rather than on the basis of an average, we shall find that from 152 to about 200 it increases in amplitude. After 200 it shows a slow falling off up to a frequency of 275, then a great diminution until the frequency of the chord-tone reaches 500 where the influence of the lowest resonance region begins to be strongly felt. Above that it is of course very strong.

We are now prepared, on the basis of these observations, to formulate a definition of *i*ᵉ in similar terms to those employed for the other vowels. It consists chiefly of the following elements.

1. A strong resonance, characteristic for the vowel, at a frequency of about 1,850. In general this resonance is chiefly confined to a single upper partial, and not widely distributed over adjacent upper partials. The *exact* pitch is not so important (it may indeed vary considerably) as the fact that the upper partial reinforced strongly is generally well isolated from any others that are noticeably reinforced, save of course on very high fundamentals.

2. A low region of resonance centering at about 575, somewhat lower than for *a*^{e} and *ę*. It is particularly to be noted that between this point and the upper region there is comparatively little distributed resonance, far less than for any of the more open vowels.

3. A chord-tone relatively much stronger than for the preceding vowels of the series, but as before varying in amplitude according to the pitch. Strong at 200 and above 500, but weak from 275 until the influence of the lower resonance region begins to be felt.

The resonance curve which I add for comparison (Fig. 1) is

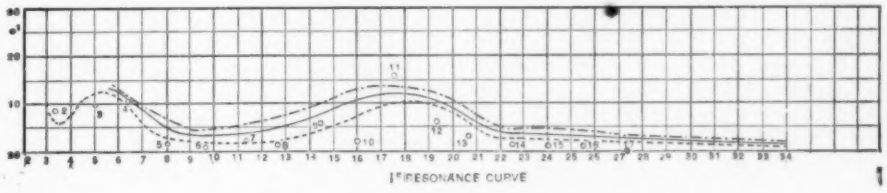


Fig. 1.

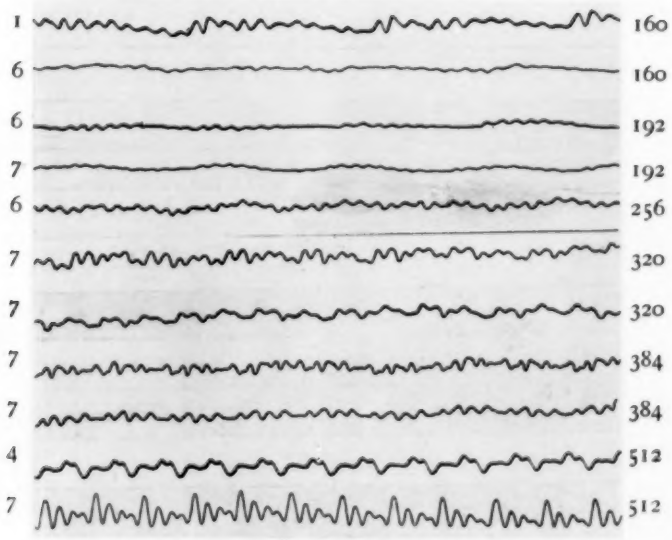
less satisfactory than the preceding ones. As an average result it fails to give a correct idea of the facts. For example, it makes the upper resonance region show a very gradual ascent and descent with a very blunt crest. This is because the individual records do not tally closely as to the exact frequency of the upper partial strongly reinforced, and averaging all together falsifies all. As a matter of fact, individual records show a steep ascent to the maximum percentage of the favored upper partial and a sharp descent beyond it. The lower region of resonance is much more stationary and the average result is therefore a truer picture. The individual record plotted on the curve, for comparison, will make these points clear.

Still the main facts are well enough indicated. The curve shows a sharp rise of resonance beginning at about 500 and a sharp de-

cline from 600, thence a low average of resonance until the upper maximum is approached. Here the rise shown is too gradual, but the maximum at about 1,850 is correct.

The examples of i^v-curves added also reveal the more conspicuous phenomena. The dominant upper partial may be distinguished at once by a simple count of the minor crests, as for example, in the long wave at 192, which shows ten minor crests ($10 \times 192 = 1{,}920$). At 320 there are six ($6 \times 320 = 1{,}920$), etc. The lower resonance is also clear in many examples, as at 320, where II. is manifestly strong ($2 \times 320 = 640$).

One of the examples at 384 is introduced to show an unusual variation, viz., a very high upper partial, VII. on 384 ($7 \times 384 = 2{,}688$). This is, of course, not the most important upper partial or the wave sound would have been something else than i^v, but it is important enough to enable one to count the crests, and it shows as a considerable percentage in the mathematical analyses. Notwithstanding this peculiarity, the sound was an i^v, and the i^v characteristic V. ($5 \times 384 = 1{,}920$) is far the strongest of the high upper partials. This analysis is not given in the table, because it is not typical.



1ª—Curves

PETER GUTHRIE TAIT.

By Alexander Macfarlane.

PETER GUTHRIE TAIT was born on the 28th of April, 1831, at Dalkeith, near Edinburgh, Scotland, where his father was private secretary to the Duke of Buccleuch. He was educated in succession at the Grammar School of Dalkeith, the Circus Place School in Edinburgh, and the Edinburgh Academy, at the last of which he had Clerk Maxwell for a class fellow. Equal in age and similar in genius Tait and Maxwell were drawn into close friendship. They graduated together from the Academy, and entered together the University of Edinburgh, where they took up the same classes of mathematics and physics. But while Maxwell continued his studies there for over two years, drinking deeply of philosophy and natural science as well as of mathematics and physics, Tait after one brief session left for the University of Cambridge, where at the age of eighteen he became a student of Peterhouse. His private tutor was William Hopkins, the most successful coach of his time. In 1852 Tait graduated as senior wrangler, won the first Smith's prize, was appointed mathematical tutor to his college, and awarded a fellowship.

The second wrangler and second Smith's prizeman of the same year was W. J. Steele, who was a member of the same college and an intimate friend of Tait. The two friends proceeded forthwith to prepare a treatise on the "Dynamics of a Particle"; but Steele lived to write a few chapters only. The book was first published in 1856, and has gone through a number of editions, Steele's name remaining on the title page.

In 1854 Tait was appointed professor of mathematics in Queen's College, Belfast. There he became acquainted with Andrews, the professor of chemistry and vice-president of the college, a skillful experimenter, who became his guide and master in physical manip-

ulation. He was privileged to join in Andrews' classical researches on the compression of gases, particularly of carbonic acid.

In 1853 Sir W. R. Hamilton published his "Lectures on Quaternions." In well-grounded anticipation of some wet days during a vacation tour in Scotland, Tait provided himself with a copy of the "Lectures," which he found to be fascinating reading even in fine weather. He was especially attracted by Hamilton's operator

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}$$

and he set to work to apply it to physical investigations. There were, however, to Tait's mind numerous obscure points in the method, and to elucidate these he wished to correspond with Hamilton directly. His friend Andrews wrote to Hamilton asking the favor; and Hamilton was only too glad to find a zealous and competent disciple. In this way a correspondence originated which was kept up till Hamilton's death in 1865. As the immediate outcome of his quaternion studies Tait published some papers in the *Messenger of Mathematics*, and several memoirs in the *Quarterly Journal of Mathematics*. The latter are entitled "Quaternion Investigations Connected with Fresnel's Wave-surface," "Quaternion Investigations Connected with Electrodynamics and Magnetism," and "Quaternion Investigations of the Potential of a Closed Circuit." In these papers he does not publish new truths, but applies the quaternion method to elucidate both in statement and in logical connection the results of previous investigators.

While professor of mathematics at Belfast he married Miss Margaret Porter, sister of the present master of Peterhouse—a lady who proved a genuine helpmate, taking full charge of all the affairs of the household in order that her distinguished husband might have perfect leisure for his scientific labors.

When, in 1860, Professor Forbes resigned the chair of natural philosophy in Edinburgh University, both of his distinguished pupils, Tait and Maxwell, became candidates for the appointment. For some years Maxwell had been the corresponding professor at Marischal College, Aberdeen, and his fame as a physicist was already well established; but the choice of the electors fell on the

energetic professor of mathematics at Belfast, who, in addition to his ordinary duties, was accustomed to experiment with Andrews during the day and write on Hamilton's quaternions at night. This contest, it is pleasant to find, did not diminish the friendship dating from their schooldays, which, on the contrary gained in intensity as the years rolled by. The letters and postals which passed between them were quaint to behold, full of scientific wit and wisdom, and occasionally expressed in verse.

While at Belfast Professor Tait made extensive preparations for publishing a volume of examples in quaternions, but as the method was new he decided to prefix some introductory chapters in explanation of the method. Soon after he removed to Edinburgh, an announcement of his forthcoming book appeared under the title of "An Introduction to Quaternions." Sir William Hamilton was then engaged in the preparation of his "Elements of Quaternions," and, on account of the correspondence which had passed between them, he did not like the prospect of Tait's book appearing before his own. He did not object to examples, but he wished to have the priority of publication in all matters of principle, so far as he himself had elaborated them. Tait, hearing of the situation, offered of his own accord to delay the publication of his volume until Hamilton's "Elements" should have appeared. Hamilton expected to publish within a year, but as a matter of fact the printing of the sheets of the "Elements" went on for four years longer, and was stopped only by his death in 1865. The "Elements" appeared in 1866, and Tait's volume the year after. The work then given to the public was entitled an "Elementary Treatise on Quaternions." The preliminary chapters, dealing with the theory of quaternions, have presented difficulties to many readers. This phenomenon is partly explained by the history of the volume, and especially by Hamilton's desire that Tait should confine the book to applications.

At the Scottish universities the academic students meet in the winter only for scarce six months, the medical and other professional students have a summer session in addition. In the winter session Tait lectured five times a week to the academic students, who were generally in their senior year, and numbered from 150

to 200. In these lectures he endeavored to traverse the whole range of elementary physics, but it was his aim to impart sound principles rather than dwell on minute details. The lecture always began punctually at seven minutes past eleven, and never ended till the clock struck twelve. In these lectures to undergraduate students he never obtruded his own researches; he had a conscientious desire to teach the appointed subjects. Every other Saturday there was an hour examination; at which, following a custom of his predecessor, he did not give out printed questions, nor write them on the board, but dictated them at uniform intervals of five minutes. Having propounded his question and rider, he grinned with satisfaction; when a student asked a question Tait reminded him that on Saturday he was changed from a benevolent teacher into a relentless inquisitor.

To help those students who wished to make a more thorough study of physics he instituted an advanced class; at this work he appeared to the greatest advantage. Before entering the lecture-room he glanced for a short time at his notes; thereafter he would write out mathematical equations for an hour without referring to any notes whatever. It was interesting to see the way in which he, like a master builder, could "sling" the symbols.

It had been Forbes' custom to allow a favorite pupil, such as Clerk-Maxwell, the use of the lecture apparatus for investigation; but at the time when Tait succeeded to the chair, any systematic laboratory teaching of physics was unknown in Scotland. About ten years later he followed the example of Sir William Thomson, at Glasgow, in instituting a practical class. It was Tait's idea that each student taking the class should be instructed how to use a variety of physical instruments, and then should be set to work upon some real experimental problem. It was in this way for instance that most of the work involved in the construction of the thermo-electric diagram was performed. Prior to the founding of the Cavendish Laboratory at Cambridge, the facilities at Edinburgh and Glasgow for gaining an experimental knowledge of physics, were the best in Great Britain; and this was due to the circumstance that in these twin cities of the North the chairs of physics were occupied by twin giants in physical science.

Professor Tait was not only great in intellect; he was likewise great in stature. In person he was handsome, well-built and athletic. He wore a longish beard; the hair on the top of his head had disappeared at an early date, and left exposed a massive cranium. To protect his head while lecturing, it was his custom to wear a velvet skull cap. On the streets of Edinburgh he presented a unique appearance, dressed in sack coat and soft felt hat, cane in hand, walking energetically, and at the same time thinking out some problem in mathematical physics, or it may be noticing in the liquid mud when a scavenger tried to sweep it into a heap a motion analogous to that of the particles of a gas when arranged in a column of increasing temperature upwards.

About 1860, the year of Tait's appointment at Edinburgh, he and Sir William Thomson sketched out a plan for an elaborate treatise on natural philosophy, in which all the branches of physics were to be expounded from the standpoint of the doctrine of energy. The plan contemplated four volumes; the printing of the first volume began in 1862 and was completed in 1867; the other three volumes never appeared. When a second edition of the work was called for, the matter of the first volume was enlarged, and appeared in two separately bound parts. The volume which did appear, although felt to be rather rough reading even by accomplished mathematicians such as Heaviside, has achieved a great success. It has been translated into French and German; it has educated the new generation of mathematical physicists, and it has received the compliment, than which none could be higher in the estimation of Tait, of being called "the *Principia* of the nineteenth century."

In 1864 Tait published in the *North British Review* articles on "The Dynamical Theory of Heat" and on "Energy," which were subsequently made the basis of his "Sketch of Thermodynamics" published in 1868. These articles, mainly historical, are written from the British point of view; so much is this the case that Continental physicists accused him of chauvinism. To this charge he replied that he had taken pains to be accurate; but if he had been partial, he was not writing as a judge but as an advocate, and that it would be impossible for the future historian to compile from the

works of the present day a complete and unbiased statement, unless contemporary history was written with some little partiality. It cannot be denied that chauvinism was to some extent a characteristic of Professor Tait; which, however good it may be in politics and commerce, is certainly out of place in the cosmopolitan field of science.

In the course of his first decade at Edinburgh, he contributed to the Royal Society of that city, in addition to various short notes, a series of elaborate memoirs on the quaternion method and its applications. Their titles are in order: "Formulæ Connected With Small Continuous Displacements of the Particles of a Medium," "On the Rotation of a Rigid Body About a Fixed Point," "On Green's and Other Allied Theorems" and "On Some Quaternion Integrals." In the two former memoirs he applied the quaternion method in a very elegant manner to deduce and connect known formulæ in the respective subjects; in the two latter he developed the processes of definite integration, which are applicable to quaternion symbols and not merely to scalar variables, a part of the analysis left undeveloped by Hamilton. For these latter memoirs the Society awarded him a Neill prize.

At the meeting of the British Association held in Edinburgh, in 1871, when Sir William Thomson was president of the Association, Professor Tait presided over the section of mathematics and physics. His address was suitable to the dual character of the section, for it consisted of two parts, a plea for Hamilton's calculus of quaternions, and observations on Thomson's doctrine of the dissipation of energy. In his plea for quaternions he pointed out the importance in mathematical physics of the operator ∇, of the linear and vector function, and of the rotation operator.

In 1873 he published in conjunction with Professor Kelland "An Introduction to Quaternions," an entirely different book from his own "Elementary Treatise." Kelland was the professor of mathematics in Edinburgh University, and it was his custom to expound to his senior class the elements of quaternions along with advanced algebra. Tait, so far as I know, never taught quaternions at Edinburgh, but in his later years he delivered several inspiring addresses on the subject to the Mathematical and Physical Societies. The book in

question grew out of Kelland's lectures; Tait revised it and added a chapter on the linear and vector function. Kelland was much the older man, and had stood to Tait in the relation of instructor. In the preface to the book he refers to the circumstance of his pupil in youth becoming his teacher in riper years; for it was Tait's papers in the "Messenger of Mathematics" which first explained to him the essence of Hamilton's invention.

Tait had now coöperated with Steele in "Dynamics of a Particle," with Sir William Thomson in "A Treatise on Natural Philosophy," and with Kelland in "An introduction to Quaternions"; there was still another literary partnership to follow. This time it was with Balfour Stewart, professor of physics at the Owens College, Manchester, a philosopher whose imagination as described by Tait himself was rather weird and grotesque. In 1875 a book entitled "The Unseen Universe; or Physical Speculations on a Future State" appeared anonymously; but to a physicist it was evidently inspired by Stewart's "Conservation of Energy" and Tait's "Sketch of Thermodynamics." It was immediately asserted in the *Academy* that Tait and Stewart were the authors; in a subsequent edition their names appeared on the title page. It was to many people a surprise that one who had been prominent in denouncing metaphysics should turn out to be part author of a book described as physical speculations on a future state. Did not Kant say that the three great problems of metaphysics are God, freedom and immortality? The kernel of the book is contained in an anagram which was published in *Nature* for Oct. 15, 1874, and which interpreted reads—"Thought conceived to affect the matter of another universe simultaneously with this may explain a future state." This other universe is the unseen universe, which bears to the luminiferous ether the same relation which the latter bears to ordinary matter; it forms the substance of the human soul which is connected with coarse matter during life, and is detached with memory complete at death. By elaborating these ideas the authors considered that they had shown the physical possibility of the immortality of the soul.

In 1876 Professor Tait published a volume entitled "Lectures on Recent Advances in Physical Science." These lectures, originally delivered to an audience of professional men, and edited from the

report of a stenographer, give a good idea of Tait's style in lecturing, at which he had no equal in Edinburgh University, save perhaps Sir William Turner. On reading these lectures, published 26 years ago, one is struck by the greatness of the *more* recent advances in physical science; for in them we find no mention of the telephone or microphone, the dynamo or the incandescent lamp; electric waves and cathode rays were as yet undemonstrated. The advances treated of are the doctrine of energy, spectrum analysis, the conduction of heat and the structure of matter.

When the British Association met in Glasgow in 1876, under the presidency of his friend Professor Andrews, he was invited on short notice to deliver one of the popular lectures. He took for his subject "Force," and using that term as an example of ambiguity, made a plea for the definite and accurate use of terms in physical science—a reform which has now made great progress, due in no small degree to his efforts. Another point he insisted on is that matter and energy are things, that is, have objective existence, because their quantity in the universe is constant, whereas force is not a thing as its quantity is infinite. It is only things, he said, which can be sold. In view of this dictum it is interesting to observe that some courts have held that an electric current cannot be stolen, as it is not a thing. But what is stolen is the energy of the current and according to Tait's ideas energy is a thing. In a subsequent paper contributed to the Royal Society of Edinburgh in 1882 he proceeds to a logical consequence of these ideas, namely, that Newton's laws of motion do not form the best basis for mechanical science; and he enunciates the following as the true bases: (1) The conservation of matter, (2) the conservation of energy, (3) that property of matter in virtue of which it is the necessary vehicle, or as the case may be, the storehouse of energy.

At the same meeting of the British Association Professor Tait communicated to section A, a paper on "Some Elementary Properties of Closed Plane Curves," which proved the starting point of an elaborate investigation into the theory of knots, the results of which are embodied in three memoirs printed in the *Transactions* of the Royal Society of Edinburgh. Knots are there divided into orders

according to the number of irreducible crossings, and a census is given of all the essentially different forms up to and including the order having ten crossings, to which the Rev. T. P. Kirkman and Professor Little added the census for the order having eleven crossings. These different forms are all figured. The lowest order has three crossings and but one form—the trefoil knot. In the course of this knotty problem Tait encountered ideas which he labelled "knottiness," "beknottedness," and "knotfulness" respectively, and in this field of research he had but one predecessor—Professor Listing, of Gottingen. Tait was drawn towards the subject by an anticipated application to the theory of the structure of matter. If the atoms are vortex-rings, their differences in kind, which give rise to differences in their spectra, must depend on a greater or less complexity in the structure of the ring or closed filament; and this difference would depend on the knottiness of the ring. The simplest kind of atom would correspond to the unknotted ring; the next simplest to the unique form of three crossings—the trefoil knot; the next simplest to the single form of four crossings, and so on. The vortex-theory of matter is not so prominent now as it was then; but, however that may be, we have in these memoirs an elegant contribution to pure mathematics, beautiful and interesting in itself, and which some day may receive a brilliant application.

In the years 1884 to 1885 he published a series of text-books on Heat, Light, and the Properties of Matter, which were designed to supplement his own lectures, and are, therefore, scarcely suitable for universal text-books. In the volume on light he gives the results of an investigation on inverted mirage, in which he applied Hamilton's general method in optics.

An important part of the scientific work of the *Challenger* Expedition was the observation of the temperature of the deep sea. The observations were made with thermometers whose bulbs were protected from increased pressure due to depth in the sea, but whose stems including certain aneurisms, that is, swellings of the bore, were unprotected. Sir Wyville Thomson, the scientific director of the expedition, was informed before starting that it had been ascertained as the result of hydrostatic experiments that the readings of these thermometers required to be corrected for

pressure at the rate of half a degree Fahrenheit for every mile under the sea. This would mean at some of the depths explored a correction of three degrees Fahrenheit, a large quantity compared with any variation of the temperature of the sea. Professor Tait was requested to investigate the matter, and he found that almost the whole of the supposed correction was due to heat produced in the laboratory experiment by causes which do not operate in deep sea observations. He found that the true correction for such thermometers without aneurism was only one-twentieth of a degree Fahrenheit for every mile of depth, and not more than one seventh in the case of those having the largest aneurism. This practical problem led to an investigation of the effect of pressure and of temperature on the compressibility of fresh water, sea water and solutions of common salt. He calculated that owing to the compressibility of water the surface of the ocean is lowered about 116 feet. Were the result of the Florentine academicians to become true, four per cent. of the land of the globe would be submerged.

When writing for his text-book on heat, the chapter on the nature of heat, his attention was arrested by certain difficulties in the kinetic theory of gases; and this led to the preparation of a very valuable series of memoirs on "The Foundations of the Kinetic Theory of Gases," published in the *Transactions* of the Edinburgh Society during the years 1886 to 1892. In these memoirs he examines critically and elaborately the theorems of Clerk-Maxwell, Clausius, Boltzman and Van der Waals, and he expands the virial equation into a formula which agrees with the experimental results of Andrews and Amagat on the isothermals of carbonic acid.

Tait's mode of life at Edinburgh assumed what may be called in the language of his conduction of heat experiments "a steady periodic state." He travelled little, and was not easily induced to break in upon his routine. For many years it was his custom to spend the long vacation on the links at St. Andrews, on the opposite coast of Fife; and in the interval between the winter and summer sessions he would likewise visit the famous golfing ground. Whatever he did take up—whether work or play—he pursued with enthusiasm. He was a skillful golfer, exemplying the harmony of theory and practice. He noticed the phenomena of the game with the eye of a

physicist, and detected causes of "knack" which the players themselves never dreamt of.

The phenomenon that a golf ball can be "jerked" nearly as far as it can be "driven" led him into an investigation of the subject of impact. For this purpose he constructed a kind of guillotine in which the knife was replaced by a blunt edge, and the head by a cylinder of elastic material. He found among other important results that, for one and the same body, the duration of impact is less, the more violent the impact, thus explaining the golf phenomenon above mentioned.

But the grand phenomenon in golf is the long time during which a well-driven ball remains in the air considering the slight elevation of the path at starting. The investigation of this subject led to two memoirs on "The Path of a Rotating Spherical Projectile." He reduces the phenomenon to that of the twisting ball—the golf ball in the circumstances mentioned twists upwards; and he traces the explanation of the twisting ball back to Newton, who observed the phenomenon in the game of tennis. Professor Tait found the equation to the path of a well-driven ball; and to determine the initial speed of such a ball he constructed a kind of ballistic pendulum, into which at a distance of four feet a skilled player drove a golf ball. He concluded from the experimental data that 240 feet per second is a good value. In the short flight of four feet he detected by the twisting of a tape that the ball rotated as much as one or two complete turns, thus demonstrating his explanation of the long time of flight.

In the course of one of his papers he gave 250 yards as the estimate of a maximum carry, a few days later his son Frederick drove a ball five yards further. Professor Tait's family consisted of four sons and two daughters; Frederick was, by profession, a lieutenant in the famous "Gordon Highlanders," and rose to be the champion amateur golfer of the British Islands. Such were his fame and prowess that to most people, Professor Tait, the eminent physicist and mathematician, became known as the father of Freddie Tait, the champion golfer. There is no doubt the father enjoyed the son's success immensely, for the buoyant sanguine temperament of youth remained his throughout life.

In the course of his long career Tait was engaged in many scientific controversies which sometimes took a polemical character. In attacking what he considered scientific error he had the courage of his conviction, and it was his nature to use exaggerated language, which was sometimes taken more seriously than he intended. Anyhow, a stranger introduced to him in his retiring-room at the university, found a genial and buoyant gentleman, very different from any ideal which might be imagined from reading his controversial letters or papers. As regards the students who attended his lectures, their attitude was one of universal respect and admiration; while that of his researchers can only be expressed by veneration and love.

Five years ago his health began to break down before the close of the arduous winter session; but to reëstablish it a vacation on the links at St. Andrews was sufficient. He had been endowed with a splendid physique, but it had long been his custom to remain in his library to late hours, and long hours of study and mental work eventually told upon his health.

Lieutenant Tait, the champion golfer, was ordered with his regiment to the field of action in South Africa; where the regiment suffered heavily in the operations at the Modder River, directed by the unfortunate Lord Methuen. It was reported that Lieutenant Tait had been killed; but his fate remained in doubt for six weeks. A white cross now marks his grave at Koodoosberg. The story of his life has appeared in a book written by a golfer—"F. G. Tait: a Record." The death of this generous and talented son, albeit on the field of honor, was a serious blow to Professor Tait, already in failing health. Early last year, becoming unable to discharge the duties of his chair, he sent in his resignation; but his friends hoped that, freed from the strain of teaching duties, his health might recover.

His researchers immediately took steps to prepare an illuminated address to be presented in April, the date of his retirement. It is surrounded by designs, emblematic of his scientific victories. There is a scroll on which are inscribed quaternion theorems of integration; a picture of Newton, emblematic of the "Principia" of the nineteenth century; a thermo-electric diagram; inverted mirage and a

Challenger thermometer; a metal bar and thermometers arranged for Angstrom's method; the gun used in the pressure experiments; the impact guillotine; the golf ballistic pendulum; while a U tube and a radiometer symbolize joint researches with Andrews and Dewar. A profusion of knots intertwine among these emblems, while various curves drawn from his memoirs head the scroll. The address itself expresses the regret of those signing at his resignation; their appreciation of the great work which he had done as a teacher, an investigator, a writer and a moral force; and their hope that health would be given him in his leisure to continue his scientific researches. There are sixty-three signatures, among them—Sir John Murray, director of the Challenger Expedition Publications; Professor MacGregor, his successor at Edinburgh; Professor Ewing, Cambridge; Professor Marshall, Kingston; Professor Scott Lang, St. Andrews; Professor Knott, Edinburgh; Professor Michie Smith, Madras; D. A. Stevenson, brother of the famous Louis Stevenson; R. T. Omond, meteorologist; H. R. Mill, geographer; Dr. Peddie, University of Edinburgh; Professor Ditrey Thompson, Dundee; Professor Crichton Mitchell, India. As the parties to the address were scattered over the whole world, its preparation took time and it was not ready for presentation in April.

Early in July Professor Tait was invited by Sir John Murray to spend some days at his home, Challenger Lodge, on the seashore near Edinburgh. He had not been there many days when he suddenly took ill, and died in the course of a few hours. The day prior to his death he spent in the garden, where he filled a sheet of foolscap with a quaternion investigation. He had often looked forward to devoting the period of his retirement to quaternion research, but that period when it came embraced only three brief months.

Professor Tait was for many years the General Secretary of the Royal Society of Edinburgh, and in that capacity he did herculean work for the Society. Belfast conferred on him the honorary degree of D. Sc., and Dublin elected him an honorary fellow of the Royal Irish Academy. The Cambridge University Press undertook the reprinting of his scientific papers. Two volumes appeared under his own supervision; a third and concluding volume

is in preparation. Looking at the memoirs in these two handsome volumes and the contributions to science contained in his separate works, we cannot help thinking that the scientific honors conferred on him in his lifetime were not at all commensurate with his greatness either as a mathematician or as a physicist. The illuminated address, too late to be presented to himself, was presented to his widow; and a movement is now on foot at the University of Edinburgh to raise a memorial worthy of and appropriate to his scientific achievements—a Tait physical laboratory.

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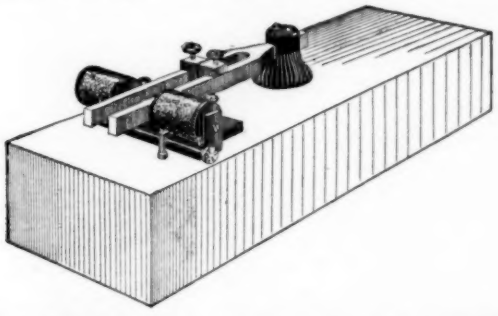

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Henry Augustus Rowland JOSEPH S AMES

Determination of the Wedge Constant of a Stellar Photometer . . . J. A. PARKHURST

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On a New Form of Interferometer CH. FABRY and A. PEROT

Minor Contributions and Notes: Variability in Light of Eros, EDWARD C. PICKERING; Variation in Light of Eros, EDWARD C. PICKERING; *Nova Persei*, WILLIAM H. PICKERING; *Nova Persei* February 28 to April 4: Stonyhurst College Observatory, WALTER SIDGREAVES.

Reviews: Annals of the Astrophysical Observatory of the Smithsonian Institution, S. P. LANGLEY (G. E. H.), 280.

JUNE, 1901.

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Minor Contributions and Notes: The Publications of the Alleghany Observatory, F. L. O. WADSWORTH; Spectrum of *Nova Persei* No. 2, EDWARD C. PICKERING; *Nova Persei* No. 2, EDWARD C. PICKERING.

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Nebulosity about *Nova Persei* G. W. RITCHEY

Notes on Variable Stars . . J. A. PARKHURST

The Wave-Number System of Rhodium C. P. SNYDER

Spectroscopic Studies. I. EDUARD HASCHEK

Minor Contributions and Notes: Observations of the Spectroscopic Binary η Pegasi, W. W. CAMPBELL; The Orbit of the Spectroscopic Binary η Pegasi, RUSSELL TRACY CRAWFORD; Opposition of Eros (433) in 1903, EDWARD C. PICKERING; The Inferior Conjunction of Mercury on November 4, 1901, F. RISTENPART; Publication of a New Catalogue of Variable Stars, MESSRS. DUNÉR, HARTWIG, MÜLLER, OUDEMANS; A Correction, H. SEELIGER; Concerning the Paper "On the Absorption Spectrum of Chlorine" ELIZABETH R. LAIRD.

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LXXIV. April, 1902

TABLE OF CONTENTS

The Graphic Root of Solutions. III. A Form of the Pizzetti Calorimeter. H. F. Moore.

The Absorption Optical Properties of Asphalt. E. L. Nichols.

The Speed of Light. Louis Bevier, Jr.

The Velocity of the Ions from Hot Platinum Wires. C. D. Child.

Another Form of Cavendish Balance. G. K. Burgess.

LXXV. May-June, 1902

TABLE OF CONTENTS

The Value of the Gravitation Constant. G. K. Burgess.

The Velocity of the Ions from Hot Platinum. II. C. D. Child.

Apparatus in the study of the Induction Coil. J. E. Ives.

Optical Glass. A. W. Wood.

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